396

VOLUME 80 SEPARATE No. 396

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

JANUARY, 1954

DISCUSSION
OF PROCEEDINGS - SEPARATES

185, 223, 250,
258, 281

ENGINEERING MECHANICS
DIVISION

Copyright 1954 by the AMERICAN SOCIETY OF CIVIL ENGINEERS
Printed in the United States of America

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

Current discussion of papers sponsored by the Engineering Mechanics Division is presented as follows:

Number		Page
185	The Non-Elastic Behavior of Bridges Under Impulsive Loads (Published in April, 1953. Discussion closed)	
	Hansen, Robert J. and Archer, John S.	1
	Fraenkel, S. J. and Grinter, L. E. (Closure)	2
223	Applications of the Relaxation Technique in Fluid Mechanics (Published in July, 1953. Discussion closed)	
	Borele, Mladen	3
	Sarpkaya, Targut	10
250	Numerical Analysis of Frames With Curved Girders (Published in August, 1953. Discussion closed)	
	Conwell, William A.	15
258	Mechanics of Manifold Flow (Published in August, 1953. Discussion closed)	
	Barton, James R.	19
	Carstens, M. R. and Roberson, John	21
	Escobar, Julio	27
	Hartigan, James J.	32
	Niaz, Sadiq M.	35
	Powell, Ralph W.	39
	Vennard, John K. and Dentoni, Darrhl I.	40
281	The Development of the Turbulent Boundary Layer on Steep Slopes (Published in September, 1953. Discussion closes January 1, 1954)	
	Corrections	47

Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

The Society is not responsible for any statement made or opinion expressed in its publications.

This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N.Y.

DISCUSSION OF THE NON-ELASTIC BEHAVIOR OF BRIDGES UNDER IMPULSIVE LOADS PROCEEDINGS-SEPARATE NO. 185

ROBERT J. HANSEN,[15] A. M. ASCE, and JOHN S. ARCHER,[16] J. M. ASCE.—

[15] Associate Prof. of Structural Eng., Dept. of Civ. and San. Eng., Massachusetts Inst. of Technology, Cambridge, Mass.

[16] Asst. Prof. of Structural Eng., Dept. of Civ. and San. Eng., Massachusetts Inst. of Technology, Cambridge, Mass.

The authors' paper is of considerable value to engineers concerned with the vulnerability of bridges to attack from atomic weapons. The analysis as presented is limited to the case of attack from directly above the bridge and only mentions the possibility of attack from weapons that are detonated to either side or end of the bridge. For a thorough assessment of vulnerability, a complete consideration should be made of attack situations in which the blast wave approaches the bridge from the side. In many cases this situation would be most likely to cause failure of the bridge. The procedures of analysis presented by the authors, however, can be utilized for the latter conditions.

Several factors not mentioned by Messrs. Fraenkel and Grinter deserve consideration. First—as regards the side loading condition—the possibility of sliding a bridge off its piers through failure of the anchor bolts is a mechanism of failure which deserves investigation. Second, truss members in compression that are subject to lateral loading from the air blast are particularly vulnerable if the axis of the member is normal to the direction of the air blast. The lateral-blast loading enhances the possibility of buckling of the member under its axial load.

It should be apparent that the blast-load impulse is dependent on the percentage of openings in the bridge deck as well as on the width of the deck. The load impulse transmitted to an open-deck (steel-grid) highway bridge or an open-deck railroad bridge may be so small as to leave the bridge undamaged from a nominal atomic bomb burst 2,000 ft in the air over the bridge, without regard to the height of the bridge above the ground.

The engineer should have a thorough background[17,18,19,20,21] on the subject of air-blast loading and dynamic analysis before attempting to analyze bridges for the type of behavior mentioned by the authors.

[17] "Shock Loading of Rectangular Structures," by Walter Bleakney, *Technical Report II-11*. Princeton University Press, Princeton, N. J., January, 1952.

[18] "A Simple Method for Evaluating Blast Effects on Buildings," Armour Research Foundation of Illinois Inst. of Technology, Technology Center, Chicago, Ill., July, 1952.

[19] "Wind Loads on Truss Bridges," by J. M. Biggs, *Proceedings-Separate No. 201*, ASCE, July, 1953.

[20] "Proceedings of the Conference on Building in the Atomic Age," Dept. of Civ. and San. Eng., Massachusetts Inst. of Technology, Cambridge, Mass., June, 1952.

[21] "Proceedings of the Symposium of Earthquake and Blast Effects on Structures," Earthquake Eng. Research Inst. and Univ. of California, Los Angeles, Calif., June, 1952.

S. J. FRAENKEL[22] AND L. E. GRINTER,[23] M. ASCE.—The interest of Messrs. Hansen and Archer in bridges subjected to impulsive loads is greatly appreciated. The writers are in agreement with the points cited by the discussers.

[22] Mgr. Propulsion and Structural Research Dept., Armour Research Foundation of Illinois Inst. of Tech., Chicago, Ill.

[23] Dean of Graduate School, and Director of Research, Univ. of Florida, Gainesville, Fla.

It is entirely possible that atomic explosions resulting primarily in the lateral loading of bridges may have greater military effect than explosions which occur directly over a bridge. Lateral loading would be much more likely, for example, to produce alternate types of damage to the bridge—such as rigid-body tumbling or the sliding of the bridge off the supporting pier. As it was not the writers' intent to establish a military doctrine for attack on bridges, consideration of these other possible causes of damage to bridge structures was purposely ignored.

It is evident that openings in the bridge deck will have an effect on the damage sustained. Quantitative extrapolation to open-deck bridges cannot be made from the material presented. However, there is reason to believe that the general relations derived (in terms of bridge length, width, and height above ground) will also apply to open-deck bridges.

Two items are worthy of additional emphasis. The first is the effect of longitudinal inertia—which is included in the equations characterizing bridge behavior, and which is the principal feature distinguishing this paper from the usual single-degree-of-freedom treatment of structural vulnerability. The importance of the longitudinal inertia is approximately the same as that of the rotary inertia, and it does not seem generally permissible to omit the longitudinal-inertia term from consideration. The second item relates to the failure criterion (the ratio of deflection to span length). The criterion is admittedly simple, but it does have the great advantage of lending itself to computation and of having significance (to a greater or lesser degree) for all types of bridges. It is possible that for given bridge types, and for given functional purposes, more pertinent and more specific criteria could be given. If this were done, the establishment of general rules of behavior would be rendered most difficult (if not impossible), and not even an approximate generalized treatment could be given to this subject.

It should be realized that it was the writers' purpose to develop and illustrate a convenient method of handling the bridge-vulnerability problem and to derive certain general relations applicable to all bridges. It was not their intent to provide quantitative information on the vulnerability of any specific bridge. As a result of the continually changing state of knowledge concerning blast loading, quantitative answers for specific bridges should always be obtained with reference to the most recent information.

DISCUSSION OF APPLICATIONS OF THE RELAXATION TECHNIQUE IN FLUID MECHANICS
PROCEEDINGS-SEPARATE NO. 223

MLADEN BORELE[a]—As indicated by the variety of results contained in the author's paper, the relaxation method is a powerful means for solving engineering problems. Since the method is essentially one of successive approximation, so that the process involved is of necessity rather laborious, it is of great importance to explore all possibilities for reducing the necessary time to attain a certain specified accuracy. From the experience gained in the calculation for free-surface flow toward partially penetrating wells by the relaxation method, which the writer undertook at the Polytechnic Institute of the University of Grenoble under the direction of Professor Hunter Rouse, he believes that the fineness of the mesh and hence the labor involved depend to a great degree upon the treatment of the irregular stars and the singular points.

There are three ways in which irregular stars can occur. They appear (A) in the region where a finer mesh matches with a coarser one or (B) at a fixed or free boundary, or else (C) they can simply be chosen under favorable conditions as a matter of preference. Wherever irregular stars are encountered, it is necessary to have a formula for the calculation of residues. For an irregular star having legs of different lengths as shown in Fig. 1, one can obtain from Eq. (7) of the authors' paper the formula

$$a^2\nabla^2\phi = (\phi_2-\phi_0)+\frac{\phi_4-\phi_0}{\lambda_4}+\frac{\phi_2-\phi_4}{n(1+\lambda_4)}+\frac{a}{b}\left[(\phi_1-\phi_0)+\frac{\phi_3-\phi_0}{\lambda_3}\right] \qquad (1)$$

for the residue, whereas, using Taylor's expansion, the writer obtains

$$a^2\nabla^2\phi=\frac{1}{1+\lambda_4}\left[(\phi_2-\phi_0)\left(2+\frac{\lambda_4}{n}\right)+\frac{\phi_4-\phi_0}{\lambda_4}\left(2-\frac{1}{n}\right)\right]$$

$$+\frac{a^2}{b^2}\frac{2}{1+\lambda_3}\left[(\phi_1-\phi_0)+\frac{\phi_3-\phi_0}{\lambda_3}\right] \qquad (2)$$

which, in spite of its apparently more complicated form, is no more difficult to apply than Eq. (1). A comparison of these two formulas in connection with each of the three ways in which the irregular stars can occur will show their relative merits.

A. Transition from a coarser to a finer mesh

For axisymmetric flow, the formula for the residue at the joint of a regular star is

a. Assistant, Fluid Mechanics Laboratory, Polytechnic Institute, University of Grenoble, France. On Leave, Hydrotechnic Institute, Serbian Academy of Sciences, Beograd, Yougoslavia.

$$a^2\nabla^2\phi = \phi_1+\phi_2+\phi_3+\phi_4-4\phi_0+\frac{a}{2r}(\phi_2-\phi_4)$$

$$-2\sum_{n=2}^{\infty}\frac{a^{2n}}{(2n)!}\left(\frac{\partial^{2n}\phi}{\partial r^{2n}}+\frac{\partial^{2n}\phi}{\partial z^{2n}}\right)-\frac{a}{r}\sum_{n=2}^{\infty}\frac{a^{2n-1}}{(2n-1)!}\frac{\partial^{2n-1}\phi}{\partial r^{2n-1}} \quad (3)$$

in which the authors' notations are used. The quantities at the right of Eq. (3) involving the derivatives of y will be denoted by ΔE. It is immediately clear that if these derivatives are large and r is small, ΔE is not necessarily small. Thus the greater the variation of the function ϕ and the smaller the values of r, the finer the mesh must be. This seems to have been universally recognized, for previous works have shown that the fineness of the actual mesh used varies from region to region. Since the transition from a fine mesh to a coarser one is effected by irregular stars, it is evident that if the calculation for the irregular stars is not very precise it is necessary to use a finer mesh, thus increasing the number of joints.

The case of axisymmetric flow into a confined well, for which the exact solution is known, can be used as a basis for comparison. Supposing that ϕ_{n-1} and ϕ_{n+1} in Fig. 2 are the correct values, one can calculate ϕ_n according to Eqs. 1 and 2. From ϕ_{n-1} and ϕ_{n+1} with the value of ϕ_n obtained in each case, it is possible to calculate the discharge per unit length q_1 through a cylindrical surface of radius $(n-\frac{1}{4})a$, and q_2 through a cylindrical surface of radius $(n+\frac{1}{2})a$. These discharges in general differ from the correct value q. The absolute values of the deviations of q_1 and q_2 from q as calculated from Eq. (1) are denoted by Δq_1 and Δq_2, and their mean value is denoted by Δq. The corresponding quantity calculated from Eq. (2) is denoted by $\Delta q'$. The following table shows that for a specified accuracy the value of n corresponding to Eq. (1) is larger than that corresponding to Eq. (2), so that Eq. (2) is preferable to Eq. (1).

TABLE 1

n	1	2	3	4	5	6
$100\,\Delta q/q$	11.2	6.01	4.07	3.11	2.40	2.01
$100\,\Delta q'/q$	11.2	2.65	1.20	0.72	0.43	0.33
$\Delta q'/\Delta q$	1.00	0.440	0.298	0.232	0.179	0.164

B. Boundary conditions

Irregular stars usually occur at the boundary, which may be fixed or free. For a free surface the boundary condition can only be checked after all the

residues have been annulled. If the expression for the residue is not correct, the free-surface position will not be correctly determined.

To make a further comparison of Eqs. (1) and (2), one may consider the region near the free surface in seepage flow. Beside satisfying the Laplace equation, the potential function ϕ has to satisfy the following two conditions at the free surface: (a) $\phi = z$ and (b) $\frac{\partial \phi}{\partial n} = 0$, in which z is the elevation of the free surface, and n is measured in a direction normal to the free surface. Because of (b) it is possible to consider the point A' instead of the point A (Fig. 3), thus eliminating one irregular leg and leaving only one leg of irregular length. Equation (1) gives

$$a^2\nabla^2\phi = \phi_2+\phi_4-2\phi_0+\frac{\phi_2-\phi_4}{2n}+\phi_1-\phi_0+\frac{\phi_3-\phi_0}{\lambda_3} \qquad (4)$$

which becomes

$$a^2\nabla^2\phi = \phi_2+\phi_4-2\phi_0+\frac{\phi_2-\phi_4}{2n}+\phi_1-\phi_0+a\sin^2\alpha \qquad (5)$$

since

$$\frac{\phi_3-\phi_0}{\lambda_3} = a\sin^2\alpha$$

Similarly, Eq. (2) gives

$$a^2\nabla^2\phi = \phi_2+\phi_4-2\phi_0+\frac{\phi_2-\phi_4}{2n} + \frac{2}{1+\lambda_3}(\phi_1-\phi_0+a\sin^2\alpha) \qquad (6)$$

To make a cursory comparison of Eqs. (5) and (6), one can let

$$\lambda_3 \longrightarrow 0 \quad , \quad \sin\alpha \longrightarrow 0$$

In the limit, Eqs. (5) and (6) should give directly the formula for the residue at a joint located at a horizontal impermeable bed. This is the case with Eq. (6), but not with Eq. (5). The coefficient 2 appearing in the left side of (6) shows that the direction normal to the free surface is a privileged one—a property which is not displayed by Eq. (5). Since Eq. (5) comes from Eq. (1) and Eq. (6) from Eq. (2), this means that, once again, Eq. (2) is preferable to Eq. (1).

The method of reflection used in Fig. 3 is not always convenient. One may use the ratio 2:1 for the lengths of the legs. If the joint is located on the surface, Eq. (2) gives

$$a^2\nabla^2\phi = 2\left(\phi_1+\phi_2-2\phi_0+a\sin^2\alpha - a\sin\alpha\cos\alpha+\frac{a\sin\alpha\cos\alpha}{2n}\right) \quad (7)$$

whereas Eq. (1) gives

$$a^2\nabla^2\phi = \phi_1+\phi_2-2\phi_0+a\sin^2\alpha - a\sin\alpha\cos\alpha+\frac{\phi_2-\phi_0}{n} \quad (8)$$

Since the numerators of the fractions are approximately equal, the residue calculated from Eq. (7) is twice that calculated from Eq. (8).

C. Rectangular meshes

In case the gradient of the unknown function can be judged a priori to be considerably larger in one direction than in the other, rectangular meshes can be used advantageously, with the long legs in the direction along which the gradient is small. Sometimes the number of joints involved can thus be reduced by 30 to 40 percent. Evidently a more exact formula for the residue will have here a considerable importance.

If, under favorable conditions, the mesh is made rectangular, with the longer legs twice as long as the shorter ones, Eq. (2) gives

$$a^2\nabla^2\phi = \phi_2+\phi_4-2\phi_0+\frac{1}{4}(\phi_1+\phi_3-2\phi_0)+\frac{\phi_2-\phi_4}{2n} \quad (9)$$

whereas the left side of the corresponding formula given by Eq. (1) is only half as great. The difference is, consequently, very large.

After making the foregoing comments on the formulas for the residue at the joint of an irregular star, the writer wishes to discuss a method of treating the singular points in some detail. The method involved consists essentially in the proper weighting of the legs, with the purpose of avoiding an extremely fine mesh which would otherwise be necessary near the singular points.

In Fig. 4, after the values of the potential are known at the points around A, B, and C, one can form the residues, which will be seen to differ considerably from zero. Since these differences arise largely from the legs containing the singularity, one may choose certain weights for these legs so that the residue vanishes. For the point B the difference $\phi_A - \phi_B$ must be multiplied by 0.77 instead of 1. The weights for the legs of the various joints are shown in Fig. 4.

To give an idea how the method of weighting can be applied to the calculation for a well, one considers the potential function

$$\phi = Lr^2+M\log r+N \quad (10)$$

One has, in this case,

$$a^2\nabla^2\phi = 4La^2 = (\phi_2-\phi_0)\frac{a^2}{r^2}\frac{4}{k-m\ell}-(\phi_0-\phi_4)\frac{a^2}{r^2}\frac{4\ell}{k-m\ell}$$
$$= (\phi_2-\phi_0)A_\ell-(\phi_0-\phi_4)B_\ell \quad (11)$$

in which

$$k=\left(\frac{r_2}{r_0}\right)^2-1\ ,\quad m=1-\left(\frac{r_0}{r_4}\right)^2\ ,\quad \ell=\frac{\log r_2-\log r_0}{\log r_0-\log r_4} \qquad (12)$$

For the case shown in Fig. 2, one obtains the coefficients presented in Table 2.

TABLE 2

n	1	2	3	4
A_ℓ	1.7778	1.5788	1.5031	1.4628
B_ℓ	1.7778	2.2255	2.3716	2.4443

In conclusion, the writer wishes to express the opinion that in each of the three cases in which irregular stars can occur Eq. (2) is preferable to Eq. (1). Moreover, by the use of proper weights considerable economy in labor can also be achieved in the vicinity of a singular point.

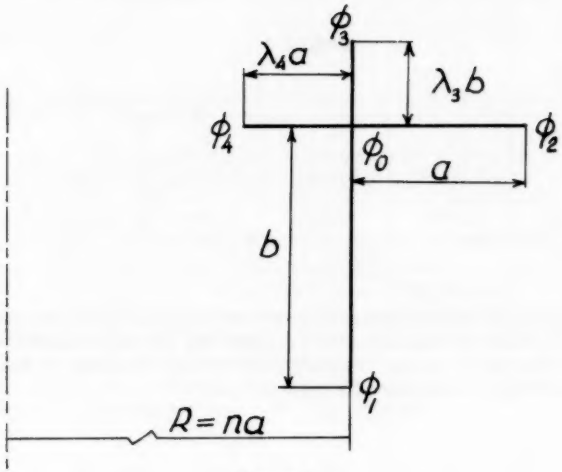


Figure 1

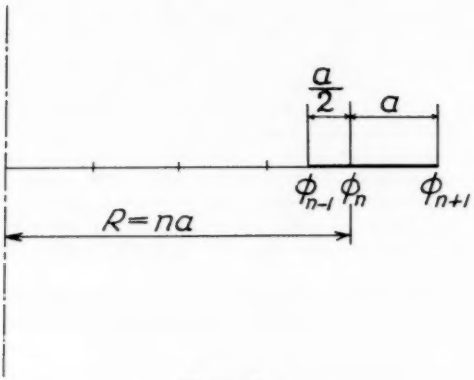


Figure 2

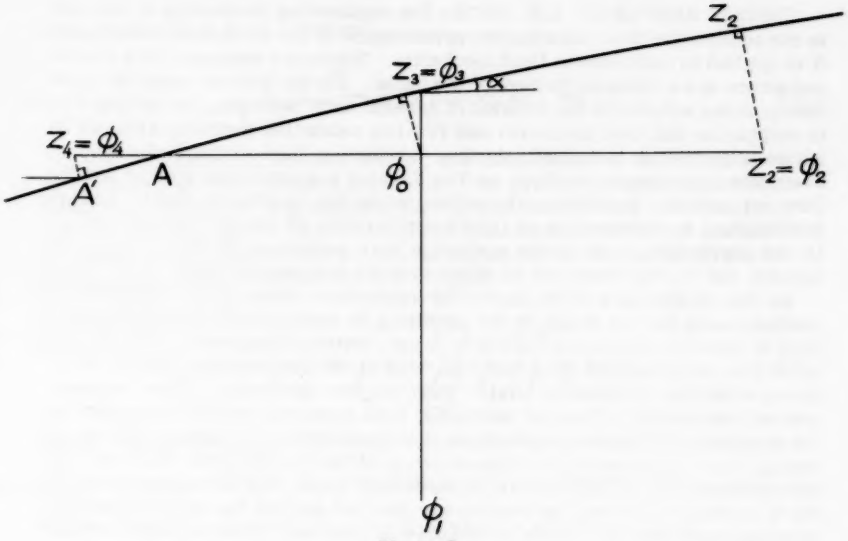


Figure 3

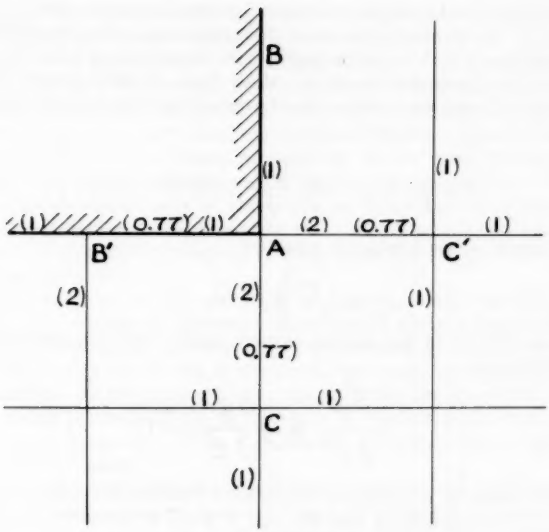


Figure 4

TURGUT SARPKAYA[b] J.M. ASCE—The engineering profession is indebted to the authors for their informative presentation of the relaxation technique as it is applied to problems in fluid mechanics. Numerous examples of a practical nature are a valuable feature of the paper. Having worked under the supervision of the authors in the solution of several such problems, the writer wishes to emphasize that both the direct and reverse relaxation methods, although by nature approximate and relatively time-consuming, have distinct advantages over such approximate methods as Von Karman's source-sink method and the flow-net method. In addition, the writer would like to enlarge slightly on their presentation by commenting on (1) the applicability of the reverse method, (2) the application of the direct method to have problems, (3) the accuracy of results, and (4) the treatment of sharp corners and singularities.

As was emphasized in the paper, the application of the direct relaxation method, using the x-y plane, to the problems in which some part of the boundary is exposed to constant pressure is not a rational approach to the answer, since it is very difficult to estimate to what extent the boundary should be changed between successive trials. However, the application of the reverse method, especially to free-jet problems, does provide a rational approach to the solution. The reverse method, as it is presented in the paper, has the advantage that the assumption of the variation of the potential with distance, i.e., the assumption of an $x(\phi)$-curve, is made only once. The relaxation procedure for y is then completed, the results are checked against one of the Cauchy-Riemann equations, and a new $x(\phi)$-curve is obtained. This procedure eliminates the necessity of assuming another entirely arbitrary $x(\phi)$ relationship. The writer's experience leads him to recommend that the x-values for the final trial be computed with a relaxation similar to that for the y-values, rather than determined directly from Cauchy-Riemann conditions.

The authors point out that the relaxation method is applicable to certain types of wave motion. An example, the case of irrotational standing waves in a channel of constant depth and variable width, will be discussed after the following explanation of the general procedure. Many types of wave motion are essentially irrotational, and the motion can therefore be defined in terms of a velocity potential.

$$\phi = \psi(x,y) \cos\frac{2\pi}{T} \cosh k(h+z) \qquad (1)$$

in which ϕ satisfies the Laplacian equation,

$$\nabla^2\phi = 0 \qquad (2)$$

Also, if the amplitude of the motion is very small, the well-known condition for the free surface is

$$\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z} = 0 \qquad (3)$$

in which z is taken as positive in the upward vertical direction. If the value of ϕ from Eq. (1) is substituted into Eq. (2), $\psi(x,y)$ is found to satisfy the criterion,

$$\psi_{xx} + \psi_{yy} + k^2\psi = 0 \qquad (4)$$

b. Research Assistant, Iowa Institute of Hydraulic Research, Iowa City, Iowa

Also, if Eq. (3) is to be satisfied,

$$\left(\frac{2\pi}{T}\right)^2 - kg \tanh kh = 0 \qquad (5)$$

The two relationships, Eqs. (4) and (5), are fundamental to this study as well as to studies of other physical problems. The goal of the analysis is the attainment of a solution that satisfies Eq. (4) and the boundary condition, $\partial\phi/\partial n = 0$. Equation (5) represents the relationship between the period of the wave, the depth of the water, and the parameter k. Equation (4) can be written in the usual finite-difference form,

$$\sum_{i=1}^{4}\left(\frac{\psi_i}{x_i}\right) + \left[k^2 - \sum_{i=1}^{4}\left(\frac{1}{x_i}\right)\right]\psi_0 = R \qquad (6)$$

Hence, with k taken from Eq. (5) by trial, arbitrary ψ values assigned, and $\sum_{i=1}^{4}\frac{1}{x_i}$ values computed, the relaxation process can be performed. After it is completed, the elevation η of the free surface is obtained from

$$\eta = \frac{2\pi}{gT}\cosh\psi_i(x,y) \qquad (7)$$

The troughs and crests of succeeding waves are obtained by connecting, respectively, locally maximum and minimum η-values.

An application of this result is presented in Fig. 1. A parabolic transition reduces the width of a rectangular, constant-depth channel to one fourth of its original width, as shown. The depth of the still water in the channel was assumed to be 0.6 ft, $k = \eta\,\text{ft}^{-1}$, and the period 0.64 sec. Figure 1 shows the instantaneous pattern of the crests of standing waves in the channel. The amplitude of the wave, also indicated on the figure, was found to be inversely proportional to the square root of the width, in accordance with Green's law [10]. If constant depth can be assumed, the relaxation method can be applied in the analysis of seiche in irregularly shaped harbors, for which standard methods of analysis fail.

The authors caution that the accuracy of a given result can only be judged by computing the velocity or pressure distribution along a boundary or other arbitrarily selected lines. However, the computation of velocities on a free boundary even from a fine mesh if the mesh size near the boundary is of the order of magnitude of the radius of curvature of the boundary will not lead to an absolute check because of the variation of λ-values in the star. If a star with a very small λ-value is to be considered, a formula for curvalinear extrapolation must be used.

The solution of certain practical problems may present very difficult boundary conditions, such as sharp corners which give rise to singularities of various types. Special treatment is essential if the function approaches infinite values near the corner. This difficulty can be overcome by the application of a somewhat more complicated method.

The solution of $\nabla^2\phi = 0$ can be expressed by the series

$$\phi = \sum A_k r^n \cos n\theta \qquad (8)$$

in which $n = \frac{\pi k}{\alpha}$ ($k = 0, \pm 1, \pm 2, \ldots$) and α is the angle of the sharp corner as is shown in Fig. 2. For $\alpha = 2\pi$, ϕ takes the form

$$\phi = A_0 + A_1 r^{1/2} \cos\frac{\theta}{2} + A_2 r \cos\theta + A_3 r^{3/2} \cos\frac{3}{2}\theta + \qquad (9)$$

Only the first four terms need be retained. The units of r can be so chosen that r = 1 at A, B, C, and D. In terms of ϕ_A, ϕ_B, ϕ_C, and ϕ_D, which are known, one obtains the coefficients A_0, A_1, A_2, and A_3. Using these coefficients, ϕ_{A1}, ϕ_{B1}, ϕ_{C1}, and ϕ_{D1} can be expressed in terms of ϕ_A, ϕ_B, ϕ_C, and ϕ_D, as follows:

$$\begin{aligned}
\phi_{A1} &= .4572\,\phi_A + .2351\,\phi_B + .2093\,\phi_C + .0992\,\phi_D \\
\phi_{B1} &= .2351\,\phi_A + .5930\,\phi_B + .0992\,\phi_C + .0728\,\phi_D \\
\phi_{C1} &= .2093\,\phi_A + .0992\,\phi_B + .4572\,\phi_C + .2351\,\phi_D \\
\phi_{D1} &= .0992\,\phi_A + .0728\,\phi_B + .2351\,\phi_C + .5930\,\phi_D
\end{aligned} \qquad (10)$$

The mathematical analysis of the above method can be found in references [11] and [12]. These are the equations used at singular points. The relaxation procedure is carried out in the usual manner, observing that Eqs. (10) hold at singular points.

The usefulness of the relaxation method in a wide variety of problems, as well as for a variety of specific boundary conditions for a specific problem, has been demonstrated.

REFERENCES

10. Engineering Hydraulics, edited by Hunter Rouse, John Wiley and Sons, 1950, Chap. X.
11. Emmons, H. W., "Numerical Solution of Partial Differential Equations," Quart. Appl. Math., 1944, Vol. 2, pp. 173-195.
12. de G. Allen, D. W., Christopherson, D. G., Fox, L., Green, J. R., Motz, H., Shaw, F. S., and Southwell, R. V., "Relaxation Methods Applied to Engineering Problems," Phil. Trans. Royal Soc. London (A), 1945, pp. 239, 367-385, 519-537.

Trough and Crest lines for Standing waves
in a channel with variable width

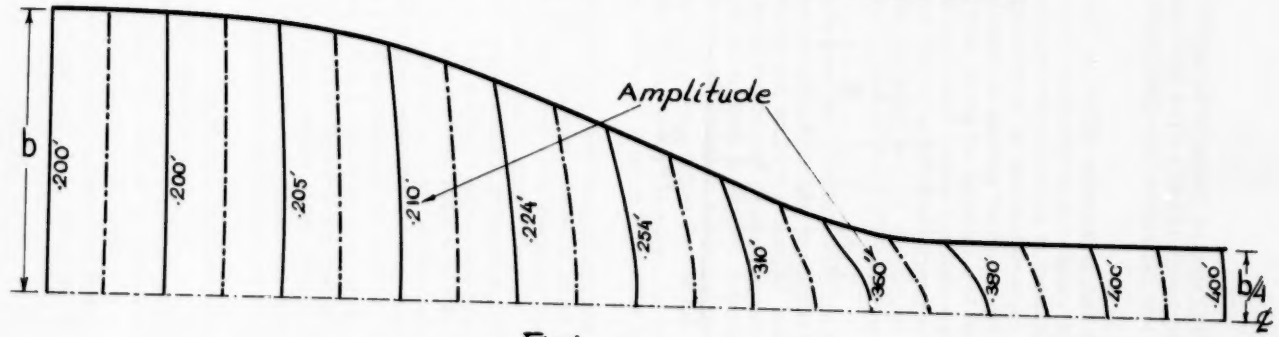


Fig. 1

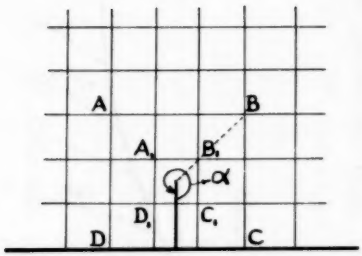


Fig. 2

DISCUSSION OF NUMERICAL ANALYSIS OF FRAMES WITH CURVED GIRDERS PROCEEDINGS-SEPARATE NO. 250

WILLIAM A. CONWELL[1], M. ASCE—A wealth of data which will be of great value to structural engineers is included in this paper. It is the purpose of this discussion, after some general comments, to indicate points which will demonstrate the wide application of the information given in the paper.

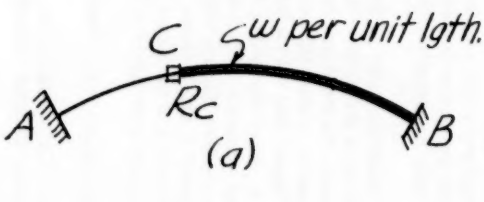
In the first paragraph under the heading "Stiffness and Carry-Over Factors" the author does not state that, during the application of the moments, the ends of the curved girders are restrained against translation although he obviously intends this. It is believed that the paper should be explicit on this point particularly in view of the current work[2] by L. E. Grinter, M. ASCE and C. H. Tsao which deals with moment distribution where translation of the joints is permitted.

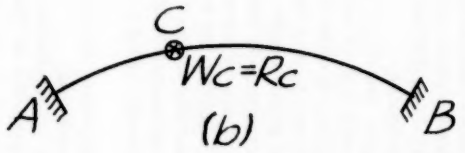
Though the author has devised excellent nomenclature for his work, the writer would prefer to see a slightly different expression for the carry-over factor. The author chooses to use the first sub-script inside the parentheses as indicating the axis to which the moment is carried; the second sub-script to indicate that about which the moment is applied. Interchanging these two sub-scripts would put the sub-scripts inside the parentheses in parallel with those outside. For instance, $(C_{XZ})_{B-A}$ would then be the carry-over factor from the X axis at B to the Z axis at A. The author, no doubt, has very good reasons for using his nomenclature. It would be of value to know what lead to its adoption.

The curves given in Figure 3 are most enlightening and are a fine representation of tabular data. The additional curves, to which the author refers under the heading "Stiffness and Carry-Over Factors," may have sufficient value to be included in the closing discussion.

One of the beauties of the paper is the fact that it permits the same procedures to be followed for circular girders that we have been in the habit of using for straight beams.[3] For instance, if one desires to obtain the fixed end moment for a uniformly distributed load on a part of the span of a circular girder one may initially solve the continuous girder shown in Figure 9(a) with a false support at C starting with fixed-end moments of Table 4. After computing the reaction at C one can, from Table 3, obtain the fixed end moments for a load W at C equal to and opposite in direction from the reaction (see Figure 9(b)). The super-position of Figures 9(a) and 9(b) yields Figure 9(c) which shows a circular girder partially loaded. It is recognized that Table 3 of the paper can be used to obtain this same information by taking a series of positions for the

1. General Engineer, Structural Engineering and Design Department, Duquesne Light Company, Pittsburgh, Pennsylvania.
2. Separate No. 298—Proceedings of American Society of Civil Engineers, October, 1953, entitled "Joint Translation by Cantilever Moment Distribution" by L. E. Grinter, M. ASCE and C. H. Tsao.
3. "Continuous Frames of Reinforced Concrete" by Hardy Cross and Newlin Dolbey Morgan, John Wiley and Sons, Inc., New York, N. Y. 1932.





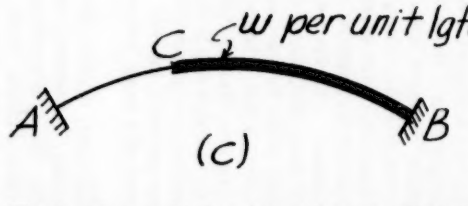
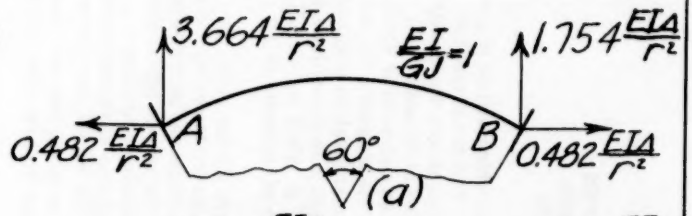
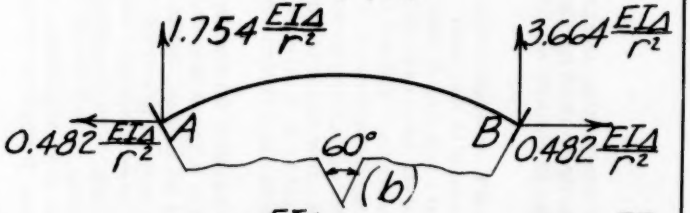


FIG. 9 : CIRCULAR GIRDER WITH PARTIAL LOAD





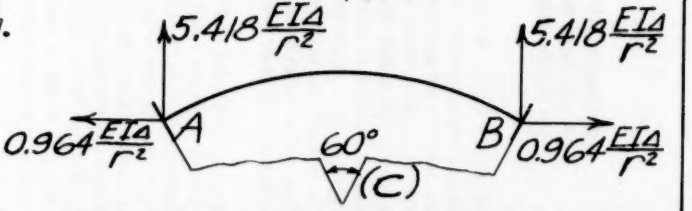


FIG. 10 : CIRCULAR GIRDER, $\phi = 60°$
F.E.M.'S FOR TRANSLATION OF JOINTS

load W. On occasions when, as stated under "Determination of Stiffness, Carry-Over Factors and Fixed End Moments," it is desired to consider shapes of girders other than circular for which data similar to the author's is not available, the procedure just outlined may result in a saving of time and effort.

When one thinks of the over-all problems of Numerical Analysis of Frames, there immediately comes to mind not only the fixed end moments, stiffness and carry-over factors mentioned by the author, but the effect of translations of the joints. In the treatment of circular girders the problem could enter in analyzing the effect of settlement at a column or perhaps, in extreme cases, for providing for the axial deformation which might take place in a comparatively light column or hanger. Although the author does not specifically mention translations of joints he has provided the data for computing this effect.

As with straight members it is apparent that the effect of translation of the joints can be obtained by introducing equal rotations at each end of a member. For instance, as shown in Figure 10, if end A of a circular girder were to settle an amount Δ the effect could be duplicated by adding the effects of rotations around Z axis introduced at ends A and B and equal to Δ/r. Figure 10 (a) shows the effect of rotating end A through an angle of Δ/r and Figure 10 (b) shows the same for an equal rotation of end B. The superposition of Figures 10 (a) and 10 (b) gives Figure 10 (c) which shows the fixed end moments due to a settlement or translation of Δ at end A, while A is held against rotation and B against both translation and rotation. These fixed end moments may be distributed as any other fixed end moments to determine the over-all effect on a structure of which member AB is a part.

The writer is pleased to have had the opportunity to discuss this paper, which he considers to be of high calibre and of great value to the engineering profession.

Corrections to Proceedings-Separate No. 250.

In Table 2, the value of the carry-over factor for EI/GJ = 1, $\phi = 60^{\circ}$, and $\eta = 0$ should be, in the column headed $K_{ZA}(C_{XZ})_{A-A}(r/EI) = K_{XA}(C_{ZA})_{A-A}(r/EI)$, equal to -0.482. In Table 5, the value of $(C_{XX})_{3-2}$ should be -1.000. In Table 6, the value of $(C_{ZZ})_{3-2}$ should be +0.487.

DISCUSSION OF MECHANICS OF MANIFOLD FLOW
PROCEEDINGS-SEPARATE NO. 258

JAMES R. BARTON.—An excellent job has been done by Dr. McNown in summarizing the basic problem of manifolds in a clear and logical manner, and engineers working on the design of projects involving manifold flow should benefit from the information presented. In a paper of such wide scope, one finds it difficult to present all of the details connected with the research even though some of them may be pertinent to an evaluation of the data. Therefore, it may be profitable at this time to briefly describe some of the experimental techniques and to evaluate the accuracy of the measurements for the experimental work on dividing flow as performed by the writer in 1946 [1].

All discharge measurements were made in weighing tanks and a minimum of two measurements were made for each discharge. A careful study of these data indicated a maximum probable error of about 1 percent. However, the measurements of piezometric head were not quite as accurate. Although capillary tubes were inserted in the lines to damp out fluctuations in both the mercury manometer and the air-water differential gage, some fluctuation still existed. A study of the piezometric head data resulted in an estimated maximum probable error of about 2 per cent with a possibility of 3 per cent error in the vicinity of the lateral where excessive disturbance occurred.

Determination of the proper hydraulic gradient for each pipe was at first a problem, since the individual piezometric head measurements showed some scatter. In order to bypass the repeating decision of "what should the exact slope be on this run?", each lateral and the main pipe were carefully calibrated by making a log plot of Reynolds number vs. hydraulic gradient. Each of these plots was based on a minimum of nine different slopes which in turn were based on a minimum of 6 point measurements of piezometric head. With these calibration plots the correct slope could always be fitted to the piezometric head data for individual runs even though the data exhibited some scatter. Of course, in the case of excessive scatter the run was discarded and repeated. A study of Fig. 3 will indicate that a change of slope of any of the hydraulic grade lines could affect the terms Δh and h'_f, especially when these terms were small. For this reason the precaution of using the correct slope for each flow measurement is believed to have avoided some errors which might have resulted in a larger scatter of the data plotted in Figs. 4 and 5.

In order to illustrate the usefulness of the author's data as a guide in research or practice, the writer would like to briefly present some findings of a Master's research project completed at the University of Utah in 1949 by Mr. Grant Borg.[1] With the object of establishing a workable design for the underdrainage system of small rapid sand filters, Mr. Borg ran extensive tests on the discharge characteristics of a perforated pipe underdrain. The research resulted in the establishment of certain relationships which must

1. "Design Data on Perforated-Pipe Underdrains for Small Rapid Sand Filters" by Grant Kenneth Borg, M.S. Thesis, University of Utah, 1949.

occur in order to give satisfactory distribution of wash water through the perforated pipe system.

The equipment consisted of one-4-inch manifold 6 feet long with 12 one-inch laterals attached to each side of the pipe on 6-inch centers so that all laterals were in a single plane. Orifices varying from 1/8 inch diameter to 7/16 inch diameter were drilled in the bottom of the laterals on 4.18 inch centers. In this way, the 4-inch pipe acted as a manifold for the lateral pipes which extended about 21 inches from the center of the 4-inch pipe, and each lateral in turn was a manifold system for the orifices which discharged into the atmosphere. Since the orifice discharge was not always at an angle of 90° to the center line of the lateral, Mr. Borg's data cannot be compared in the strictest sense with the author's data, but in some details a qualitative comparison exists which is interesting to note.

Although other comparisons might be made, only one will be included here to illustrate how the author's data would have been useful in the research at Utah. For small $\frac{D_1}{D}$ ratios, and varying $\frac{Q_1}{Q}$ ratios the flow distribution through the orifices was very uniform as Fig. 5 indicates it should be. For $\frac{D_1}{D} = 1/4$, the ratio of $\frac{h'_f}{V_1^2/2g}$ remains essentially constant over a wide range of discharge ratios $\frac{Q_1}{Q}$. However, as $\frac{D_1}{D}$ increases, the ratio of $\frac{h'_f}{V_1^2/2g}$ varies considerably over a wide range of $\frac{Q_1}{Q}$. The results of Utah clearly illustrated that as the orifice was enlarged, the flow distribution became poorer because the ratio of $\frac{h'_f}{V_1^2/2g}$ varied considerably with Q_1/Q and Q_1/Q naturally varied greatly from the first orifice on the lateral to the last one. The laboratory research at Utah was for a specific purpose, and the work was very well done, but the author's results could have been effectively employed as a guide, thereby reducing the total amount of work required for a significant answer.

M. R. CARSTENS[1] and JOHN ROBERSON[2]— Professor McNown has clearly and concisely described the mechanics of manifold flow. The results presented in this paper are a demonstration of the values to the engineering profession of wisely directed graduate-student research programs.

With the principal characteristics of the flow defined for a single branch point in a conduit, it is possible to incorporate the results into the analysis of a multiple branching conduit system. The only additional restriction to such an analysis is that the branch points be separated by a sufficient length of straight conduit. The reported experiments are indicative that a spacing of 20 diameters between branch points is sufficient. As mentioned by Professor McNown, the multiple branch analysis must be started with the farthest downstream branch and must be continued by successively accounting for the effects of each section of intervening main line conduit and of each branch. The

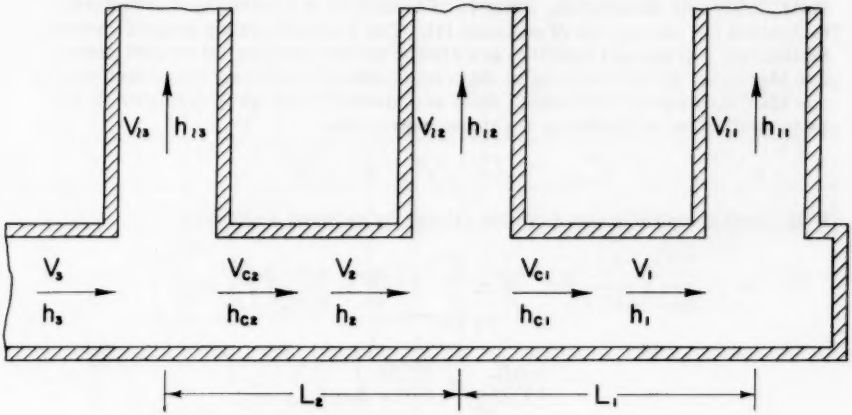


Figure 1. Definition Sketch for Multiple Branch Flow.

general expression relating the piezometric head in the upstream or n th branch to the downstream or 1 st branch (figure 1 and figures 1a and 3a, McNown) is as follows:

$$h_{in} - h_{i1} = \sum_{i=1}^{n-1} \left\{ \frac{fL}{D} - \frac{\Delta h}{V^2/2g} \right\}_i \frac{V_i^2}{2g} - \left\{ \frac{\Delta h}{V^2/2g} + \frac{h - h_i}{V^2/2g} \right\}_n \frac{V_n^2}{2g} + \left\{ \frac{\Delta h}{V^2/2g} + \frac{h - h_i}{V^2/2g} \right\}_1 \frac{V_1^2}{2g} \qquad (1)$$

1. Associate Professor, School of Civil Engineering Georgia Institute of Technology, Atlanta, Georgia.
2. Assistant Hydraulic Engineer, Division of Industrial Research and Instructor, Civil Engineering, State College of Washington, Pullman, Washington.

The parameter fL/D is the loss coefficient for the straight pipe section between the branch points. The values of $\Delta h/V^2/2g$ can be determined directly from figure 4, McNown. The value of $(h - h_1)/V^2/2g$ can be determined from figure 5, McNown, as follows:

$$\frac{h-h_1}{V^2/2g} = \frac{h_f'}{V^2/2g} + \left(\frac{Q_1}{Q}\right)^2\left(\frac{D}{D_1}\right)^4 - 1 \qquad (2)$$

The computation procedure indicated by equation (1) is satisfactory for analysis of an installed system but laborious trial computations are involved in the calculation for design.

In many situations the multiple branch variables are limited and the solution of equation (1) can be presented on a nomograph. Work was done at the State College of Washington, Division of Industrial Research, by the writers to develop the nomograph of equation (1). The multiple branch simplifications, facilitating a graphical solution, are chosen as follows: (a) all branch points are identical; (b) all sections of main line conduit connecting the branch points are identical; and (c) all branch lines are identical both as to diameter D_1 and as to coefficient of discharge C_d in the expression

$$C_d = Q_1/A_1\sqrt{2gh_1} \qquad (3)$$

With these simplifications equation (1) can be reduced as follows:

$$\left(\frac{1}{C_o}\right)^2\left(\frac{D}{D_1}\right)^4\left(M_n^2 - 1\right) = \sum_{i=1}^{n-1}\left(\left\{\frac{fL}{D} - \frac{\Delta h_i}{V_i^2/2g}\right\}\left\{\sum_{i=1}^{n-1} M_i\right\}^2\right) - \left\{\frac{\Delta h}{V^2/2g} + \frac{h-h_1}{V^2/2g}\right\}_n \left\{\sum_{i=1}^{n} M_i\right\}^2 + \left\{\frac{\Delta h}{V^2/2g} + \frac{h-h_1}{V^2/2g}\right\}_1 \qquad (4)$$

M_i is the ratio of the discharge of the i th branch to that of the downstream branch, i.e., $M_i = Q_i/Q_1$. Thus M_n is the ratio of the discharge in the upstream branch to the discharge of the downstream branch.

Equation (4) has been solved for various values of the dimensionless parameters C_d, fL/D, D/D_1, and n. Figures 2 and 3 are graphical presentations of these solutions in which

$$M = \phi\left(C_d, \frac{fL}{D}, \frac{D}{D_1}, n\right)$$

and

$$\frac{\Sigma M}{n} = \phi\left(C_d, \frac{fL}{D}, \frac{D}{D_1}, n\right).$$

The value $\Sigma M/n$ is the ratio of the average discharge of all branches to the discharge of the downstream branch, that is, Q_{ave}/Q_1. The nomograph is used by following horizontally or vertically from curve to curve of known

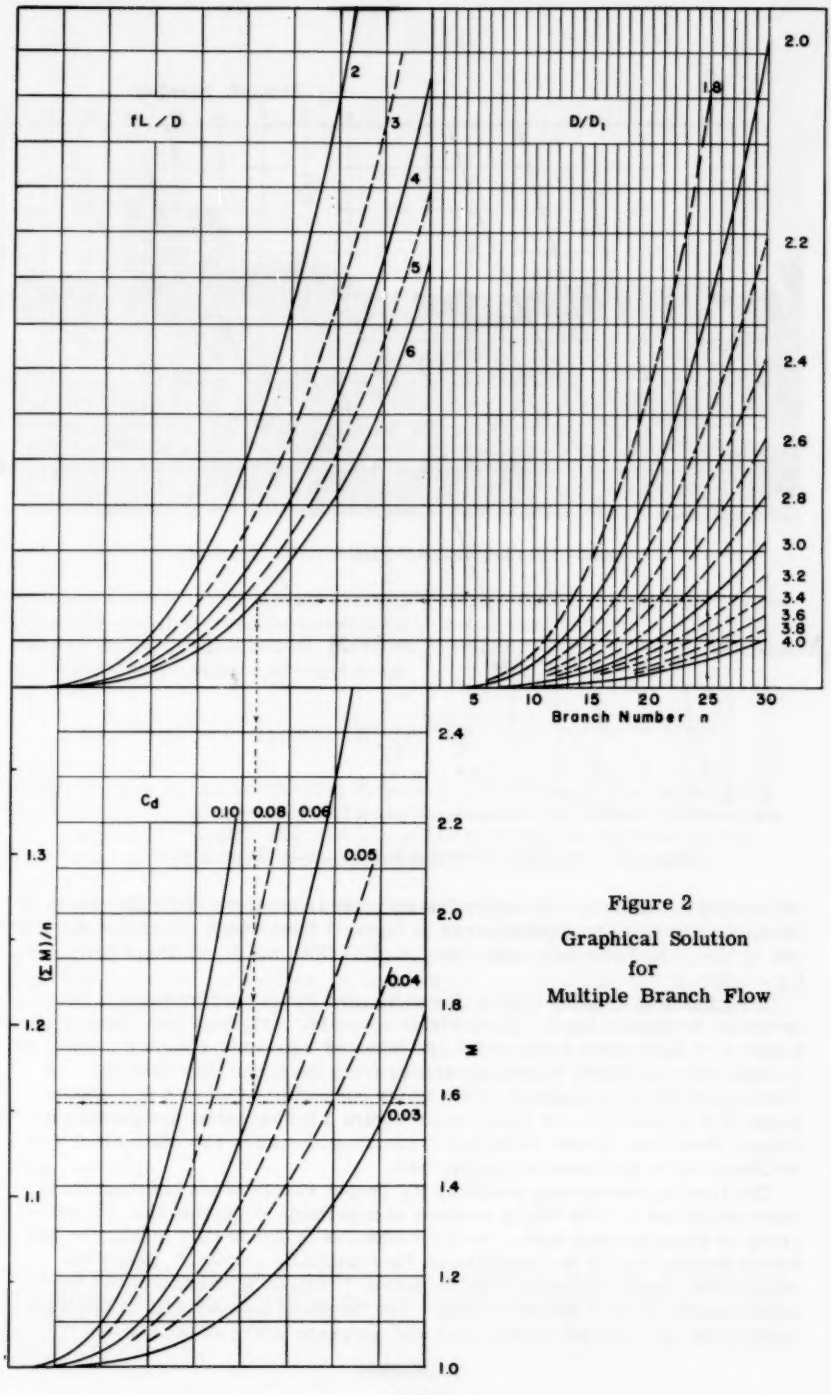


Figure 2
Graphical Solution
for
Multiple Branch Flow

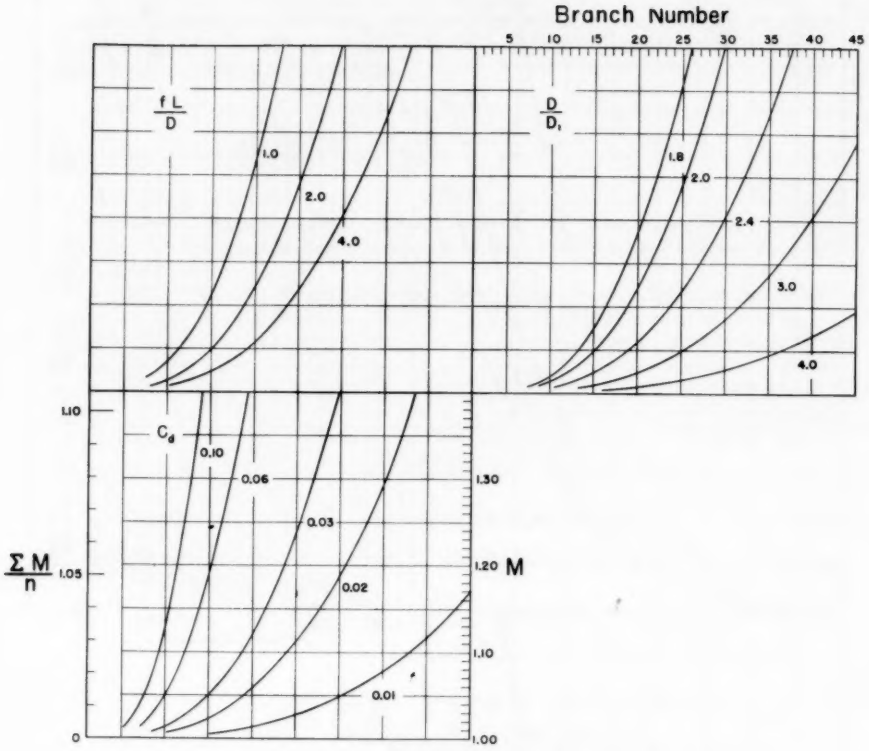


Figure 3. Graphical Solution for Multiple Branch Flow.

value until the scale or curve of unknown value is reached. This procedure is illustrated by the light dashed curve on figure 2 from which the values $M = 1.58$ and $\Sigma M/n = 1.16$ are determined for $n = 25$, $D/D_1 = 3.0$, $fL/D = 6$ and $C_d = 0.06$.

The graphical solution (figure 3) can be used for the design of portable sprinkler irrigation lines. The portable sprinkler irrigation line consists of a series of light metal tubes which are inserted into quick-coupling connectors. A riser tube (generally 1-inch) protrudes from the top of the connector. A rotating sprinkler is mounted on top of the connector. Figure 4 is a photograph of a typical line. In order to use figure 3 for sprinkler irrigation line design, there are certain additional approximations and restrictions that will be discussed in the following paragraphs.

The first approximation concerns the proper choice of the constant fL/D curve on figure 3. The tubing sections of a portable irrigation line are generally of extruded aluminum. As the roughness height is very small, the new tubing sections would be classified as "hydraulically smooth." Hence the value of the Darcy-Weisbach friction factor f will be a function of the Reynolds number of each tubing section. The Reynolds number will be the least value in the downstream section and will increase successively to the

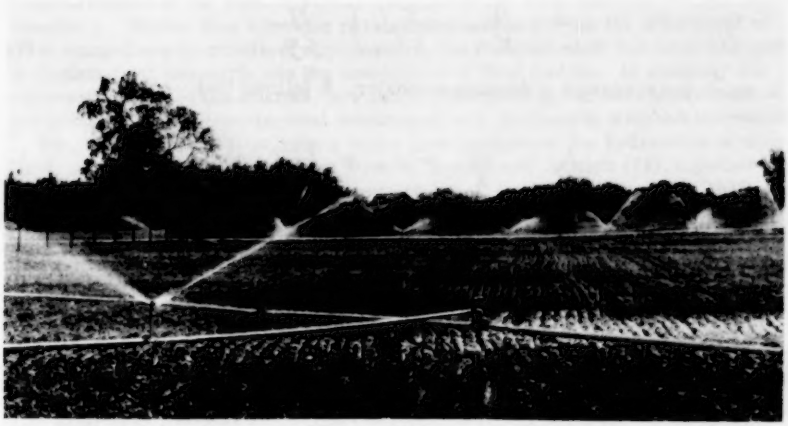


Figure 4. Portable Sprinkler Irrigation Line.

greatest value in the upstream section. The best value of a constant f to replace the variable $\underline{f}$ can be determined by a summation of the $(fL/D)(V^2/2g)$ terms. From this summation, the average velocity V_R to be used in calculating the Reynolds number is found to be

$$V_R = V_1 \sqrt{\frac{n(2n-1)}{6}} \tag{5}$$

Thus, one can determine the best value of the Reynolds number by using V_R. By referring to standard curve of $\underline{f}$ versus Reynold's number, the best value of $\underline{f}$ can be determined. The parameter fL/D is then computed using this best value of $\underline{f}$ along with the properties of the repeating tubing sections L and D.

In sprinkler irrigation line design the use of the sprinkler coefficient of discharge is much simpler than the use of the branch line coefficient discharge. In equation (1) the difference $h_{ln} - h_{l_1}$ was expressed in terms of flow and geometric properties. In equation (4) the values of h_{l_n} and h_{l_1} were eliminated, as previously explained, by the use of the branch discharge equation, that is,

$$h_l = \left(\frac{1}{C_d}\right)^2 \frac{V_l^2}{2g} \tag{6}$$

Thus the value

$$h_{ln} - h_{l1} = \left(\frac{1}{C_d}\right)^2 \left(\frac{V_{1n}^2}{2g} - \frac{V_{11}^2}{2g}\right)$$

was used in the derivation of equation (4). However, in the case of sprinklers, the discharge is a function of the sprinkler geometry and the pressure at the base of the sprinkler

$$\frac{P_s}{\gamma} = \left(\frac{1}{C_{ds}}\right)^2 \frac{V_l^2}{2g} \tag{7}$$

in which the subscript s denotes sprinkler. It follows that

$$h_l = \frac{P_s}{\gamma} + z_s + \frac{fL_l}{D_l} \frac{V_l^2}{2g} \tag{8}$$

in which z_s is the elevation of the sprinkler base and the last term is the friction energy loss in the riser. The use of the sprinkler coefficient of discharge C_{ds} in place of the branch coefficient of discharge C_d is permissible if $h_{l_n} - h_{l_1} = P_{sn}/\gamma - P_{s_1}/\gamma$. This condition is satisfied in a level irrigation line with equal-length risers and if the friction energy loss difference in the risers is negligible in magnitude. This difference is negligible for a number of reasons but primarily because this energy loss is small in relation to the value of P_d/γ used in irrigation lines.

In summary, the results as reported by Professor McNown have been incorporated into a graphical method for the solution of multiple branching conduit systems. Nomographs (figure 2 and 3) have been prepared for systems in which, (a) all branch points are identical, (b) all sections of main line conduit, connecting the branch points are identical, and (c) all branch lines are identical as to diameter D and coefficient of discharge C_d. Methods of utilizing the nomographs were discussed for the design of a level portable sprinkler irrigation line with equal-length risers. This discussion included the method of determining the best value of f to be used in the parameter fL/D and the feasibility of utilizing the sprinkler coefficient of discharge C_{ds} in lieu of the branch line coefficient of discharge C_d.

The results, as reported by Professor McNown, are deficient in respect to the minimum length of branch line. The analysis and experimental studies were predicated on the concept that the length of branch line was sufficient so as to have no influence upon the expansion of the jet (figure 2, McNown). It would seem possible to determine this minimum length from a study of the piezometric head line in the branch conduit utilizing the data obtained by Barton [1]. Professor McNown also fails to indicate why this data was not utilized in determining the minimum spacing between branch points in the main conduit.

JULIO ESCOBAR,[1] J. M. ASCE— The paper presented by Mr. McNown is representative of the kind of work conducted at the Iowa Institute of Hydraulic Research. Rather than consider particular problems from the standpoint of their immediate or practical applications, the Institute staff has been engaged in fundamental research into the mechanics of fluid motion. In evolving the different phases of the studies, new basic principles are given coherent expression and their experimental foundations are thoroughly weighed and valued.

Mr. McNown undertook such a basic investigation of the hydraulics of manifolds after the publication of the work of Soucek and Zelnick (11), together with his discussion thereof which merited the J. C. Stevens Award in 1946. After five years of research, Mr. McNown has published his findings. Faithful to the orientation of his Institute, he has presented not a series of graphs, tables, or charts applicable to the design of a particular type of manifold, lock, or water-supply system, but rather an interpretation of the behavior of flow in the vicinity of laterals, together with a definition of the general principles governing the hydraulics of manifolds.

Among the many aspects that can be studied in the problem, the one pertaining to the explanation of the energy changes and the head losses that occur in the region of the lateral for dividing flow is very important. The writer was concerned with this phase of the investigation while at the University of Iowa, and wishes to contribute additional findings from his studies (3) which are believed to be helpful in the understanding of certain phases of the phenomenon. These subjects had been discussed briefly by other investigators (1, 5, 11), particularly in regard to the presence of negative losses in the region of the lateral.

The apparent increase in the energy of flow past the lateral for small discharge ratios (lateral to total) is explained by advancing the hypothesis that the lateral discharge comes from a region of low-velocity flow, containing below-average kinetic energy relative to the remaining flow. Therefore, the equation of energy, from which the equation

$$\frac{\Delta h}{V^2/2g} = \frac{Q_l}{Q}\left(2 - \frac{Q_l}{Q}\right) - \frac{h_f}{V^2/2g} \qquad (1)$$

was established by Mr. McNown in his paper, should be modified to include velocity correction coefficients[2]

$$K_b \frac{V^2}{2g} = K_c \frac{V_c^2}{2g} + \Delta h + h_f \qquad (a)$$

with reference to Fig. 1.

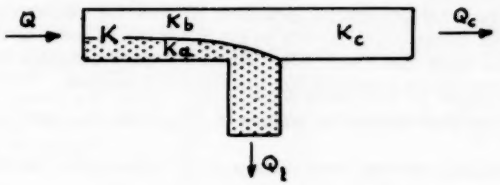


Figure 1

1. Assistant Engineer, R. J. Tipton y Asociados de Colombia Ltda., Bogota, Colombia.
2. Rouse, Hunter, Elementary Mechanics of Fluids. New York, John Wiley and Sons, Inc. 1946 - p. 114.

If the velocity correction factors are redefined as:

$$K_b = 1 + \varepsilon_1 \qquad \text{and} \qquad K_c = 1 + \varepsilon_c$$

and if Eq. (a) is rewritten in the form

$$\Delta h + \varepsilon_c \frac{V_c^2}{2g} - \varepsilon_1 \frac{V^2}{2g} = \frac{V^2}{2g} - \frac{V_c^2}{2g} - h_f \qquad (b)$$

the pressure rise will be modified by two terms depending on the velocity heads and the velocity distribution upstream and downstream from the lateral if Δh_c is the corrected pressure rise,

$$\Delta h_c = \Delta h + \varepsilon_c \frac{V_c^2}{2g} - \varepsilon_1 \frac{V^2}{2g} \qquad (c)$$

and the final equation takes the form,

$$\frac{\Delta h_c}{V^2/2g} = 1 - \left(\frac{V_c}{V}\right)^2 - \frac{h_f}{V^2/2g} \qquad (d)$$

The second member of equation (d) is the same as that of McNown's Eq. 1 plotted in Fig. 4 of his paper. Therefore, the pressure rises may be computed and compared not only with the theoretical curves but with Barton's experimental curves also plotted in Fig. 4.

The remaining problem is the evaluation of the velocity correction factors, which are defined by the equation

$$K_e = \frac{1}{A} \int \left(\frac{v}{V}\right)^3 dA \qquad (e)$$

Their evaluation depends on the manner in which the main flow is divided in the region of the lateral. Once the section of the lateral discharge is known, the energy factors can be determined and applied to the general energy equation.

It was necessary to observe in the laboratory what part of the cross section of the main pipe contributed flow to the lateral. A lucite "T" section, 2 inches by 1 inch, was inserted in the experimental 2-inch pipe to make possible observation of the path followed by dyed droplets or by air bubbles injected at various points in a section well upstream. As the use of dye was somewhat troublesome because prolonged operation colored the recirculating supply and interfered with other experiments in the laboratory, most of the exploratory measurements were made with air. However, later analysis showed that the dye solution was more accurate for the intended purpose.

Experiments showed that for values of $\frac{Q_1}{Q} < 0.5$ the area for Q_1 was bounded by slightly curved lines convex to the side of the lateral as shown in Fig. 2. For $\frac{Q_1}{Q} > 0.5$ the experiments did not show exactly the form of the section but it seemed that the bounding curves increased their curvature to form almost closed boundaries as shown in Fig. 2.

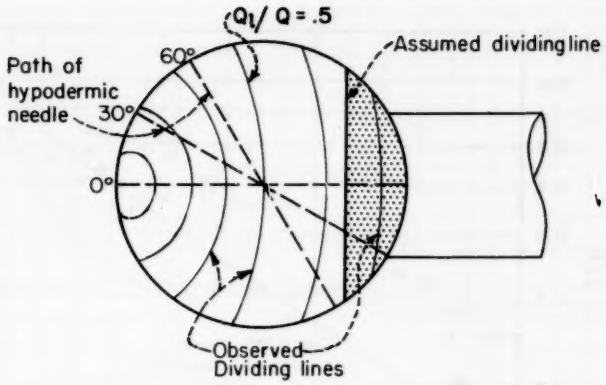


Fig.2.—LINES OF DEMARCATION BETWEEN LATERAL AND CONTINUING FLOW FOR THE FLOW APPROACHING THE LATERAL.

In Fig. 3 are shown the results obtained from the tests with air and dye for a horizontal traverse of the hypodermic needle used for dye injection; the line shown was computed from the assumption, made for simplicity, that the area required for a given lateral discharge was limited by a vertical line as shown in the figure. This assumption was not exact but proved very helpful for the interpretation of results. Other traverses were made along lines inclined 30^0 and 60^0 to that for Fig. 3 as indicated in Fig. 2.

It was mentioned before that air bubbles did not define precisely the boundary of the diverging flow as is borne out by the excessively high position of the points from the observation with air bubbles in the plot. This may be explained from a consideration of the mass accelerations involved. In the region of the lateral, the pressure gradient that produces the lateral discharge causes a larger acceleration of the injected air bubbles than of the surrounding water because of the different densities. Although this relative acceleration is diminished by the resistance and virtual mass of the surrounding medium (the principle of the virtual mass is presented in most treatises on Hydrodynamics[3,4,5] the bubbles are carried toward the lateral pipe more rapidly than the water or than the dyed droplets, the density of the latter being practically equal to that of the water. Consequently, the air bubbles do not follow the stream lines of the liquid, and, being displaced more than the fluid particles, the section of the lateral discharge determined from their paths appears to be larger than it should be.

With the assumed form of the area for the portion of Q which is diverted to the lateral, and with the assumption of a velocity distribution described by the von Karman equation[6]

3. Lamb, Horace, Hydrodynamics. New York: Dover Publications, 1945, p. 124.
4. Green, S. L. Hydro and Aero-dynamics. London: Sir Isaac Pitman and Sons, Ltd. 1937, p. 114.
5. Streeter, Victor L., Fluid Dynamics. New York: McGraw-Hill, 1948, p. 64.
6. Rouse, Hunter, Elementary Mechanics of Fluids. New York, John Wiley and Sons, Inc., 1948, p. 64.

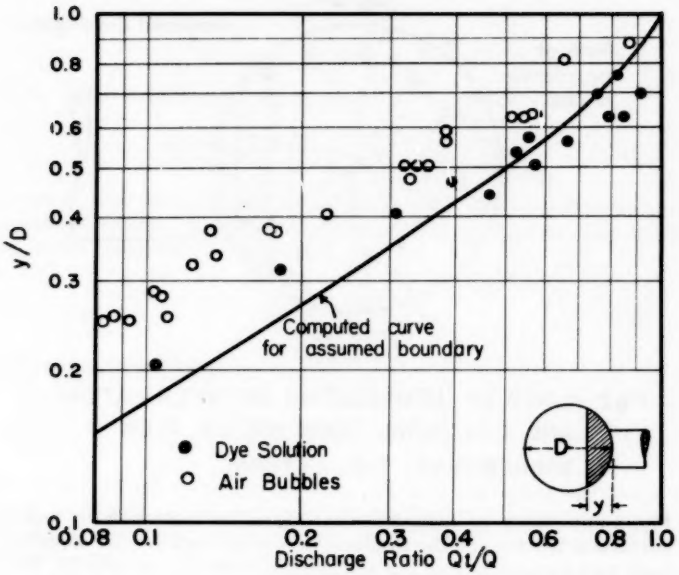


Fig.3.— EXPERIMENTAL DETERMINATION OF THE LATERAL FLOW REGION:—For traverse angle of 0°.

$$\frac{v}{V} = \sqrt{f}\,(2.15 \log_{10} \tfrac{y}{r_0} + 1.43) + 1 \qquad (f)$$

the energy factors were evaluated. From Eq. (d) it was then possible to compute corrected values of the pressure rise across the lateral, and compare computed values with those of Barton's experiments.

For $\frac{Q_1}{Q} < 0.5$ all recomputed points showed that the energy gain previously found had been accounted for. For $\frac{Q_1}{Q} > 0.5$ all recomputed points were abnormally high, indicating a failure in the assumption of the form of the area of Q_1 as has been explained before. However, some rise of the experimental points is to be expected, because the water going to the lateral has above average kinetic energy. Although it would have been possible to compute the correct pressure rise following the exact flow pattern, it was thought that the refinements obtained would not have justified the work involved in the computations since high values of $\frac{Q_1}{Q}$ are not common in practice.

The apparent anomaly of the energy increase across the lateral for low values of Q_1/Q was thus explained in a rational manner, supported by experimentation. The pressure rise is affected by the velocity distribution of the flow and either the theoretical curves of Mr. McNown's Fig. 4 could be raised or the experimental curves lowered.

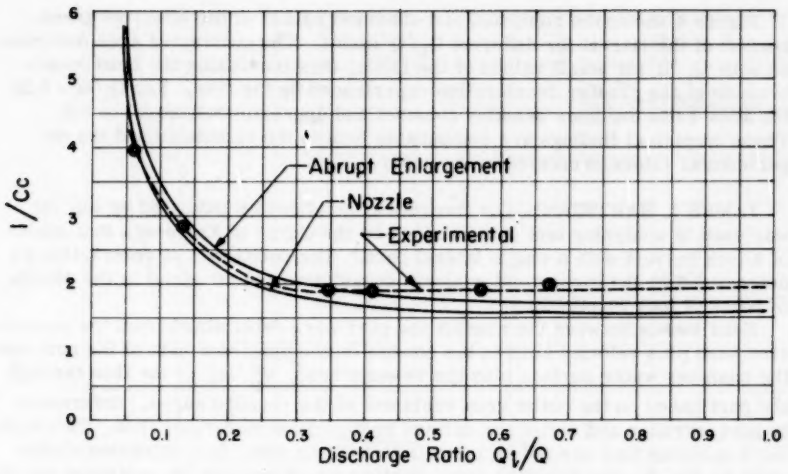


Fig.4.- COMPARISON OF OBSERVED AND COMPUTED EFFECTIVE CROSS-SECTION OF THE LATERAL.

One second phase of the problem of lateral flow to be studied from knowledge of the flow pattern is that of the form losses occurring at the entrance to the lateral. To detect the eddy formed in the region of separation of the lateral flow, another technique was developed whereby dye was injected through a hole drilled into the lateral and the extent of the colored separation zone recorded for various Q_1/Q ratios.

The reduction of the effective area of the lateral pipe due to separation of the flow as the water enters the lateral, is very similar to that occurring at an orifice plate or a flow nozzle. It seems logical therefore to compare the head losses in the lateral with those of the mentioned hydraulic transitions. Mr. McNown has shown the remarkable correspondence between the measured head losses and those based on theoretical considerations for plane irrotational flow. The writer also found good correspondence between his laboratory determination of the effective cross-section of the lateral and those computed by use of equations for the head loss for an abrupt enlargement (McNown's Eq. (5)), and for a nozzle. For the latter, the empirical equation recommended by Bean and Beitler[7]

$$h_f = \frac{V_1'^2 - V_1^2}{2g}(1 - C_c) \qquad (g)$$

was used. This equation takes the dimensionless form:

$$\frac{h_f}{V_1'^2/2g} = \left(\frac{1}{C_c}\right)^2 - \frac{1}{C_c} + C_c - 1 \qquad (h)$$

7. Bean, H. S. and Beitler, S. R., "Research on Flow Nozzles" Transactions ASME, Vol. 60, April 1938, p. 244.

Figure 4 shows the computed and observed values of the effective cross-section of the lateral for different Q_1/Q ratios. The contracted area decreases with Q_1/Q for small values of the latter, thus increasing the head losses because of the greater deceleration experienced by the flow. For $Q_1/Q = 0.35$ the area-ratio becomes sensibly constant and approximately equal to 0.5. These empirical findings are comparable both to the calculated and the experimental values presented in the paper.

JAMES J. HARTIGAN[1]—The theoretical relationship indicated by Eq. (4) was used in analyzing test data obtained by the Corps of Engineers in a model of a lock culvert with a single lateral port.[2] The coefficient of contraction as determined by the theoretical analysis should prove to be useful in the design of ports of lock manifolds.

Head losses between the conduit and port were determined from the piezometer head plus velocity head in the conduit on the upstream side of the port and the chamber water surface plus the velocity head, $V_p^2/2g$, of the flow through the port based on the outlet area exclusive of the rounded edges. References to port entrance and outlet are defined by direction, in divided flow. Test data for combining flow are not included in this discussion. In a definition sketch such as Fig. 3a, the chamber water surface would replace the upstream end of the piezometric pressure gradient in the lateral.

In Fig. A head losses in terms of $V_p^2/2g$ are shown for four ports, three of which had rounded entrances and exits and the other square edges. Dimensions of the ports are shown in Fig. B. Although the port in each case was located at the bottom of the conduit, thus limiting the contraction in the vertical to the top side, there seemed to be no reason for changing the areas to be used in determining the ratio of a/b in Eq. (4).

The ratio of port length to conduit width was evidently large enough so that values of C_c computed for a barrier of infinite length were applicable. However, it was observed in the model that some of the ports were too short to allow complete expansion of the jet before reaching the outlet. Test data are therefore compared with the theoretical curve for head loss in a sudden expansion and also for the condition of no expansion of the jet,

$$h'_f = \frac{V_p'^2}{2g} - \frac{V_p^2}{2g}$$

in which V_p' is the velocity of the jet. A curve midway between the ordinates of the curves for sudden expansion and no expansion is shown to help reference the test points.

The rounding of the entrance to the ports made indefinite the effective entrance area for determining the ration a/b. Dimensions for port 2 in Fig. B show the width and height used in determining the maximum effective entrance area. In addition to the upstream and top curvature, the downstream curvature to the 45-degree radius is included.

Since in three dimensional flow the contraction coefficient C_c is applicable to an area, values of $\sqrt{C_c}$ will apply to the width and height of the port. For port 2 the jet would not be suppressed in the vertical until the value of C_c reached 0.584, i.e., $\left[\frac{3.24}{4.24}\right]^2$. After determining the effective entrance area

1. Hydraulic Engineer, Corps of Engineers, St. Paul District.
2. Hydraulic Laboratory Report No. 52, St. Paul District, Corps of Engineers, June 1946.

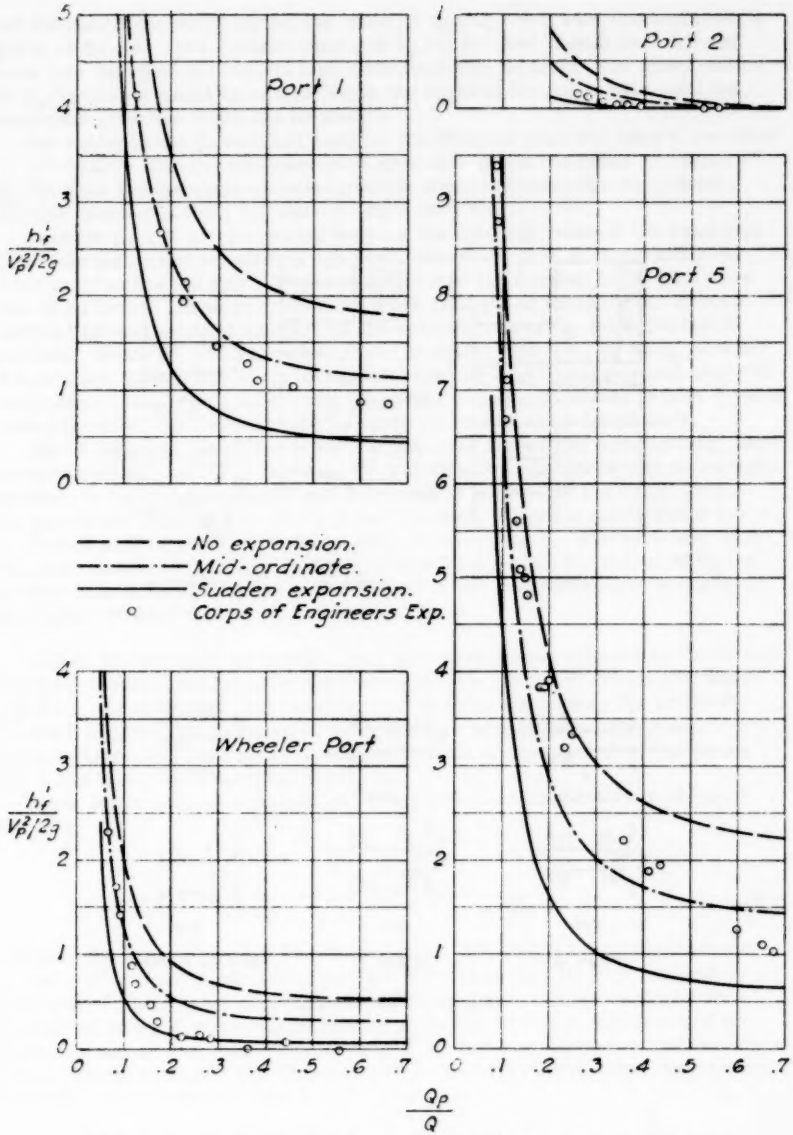


Fig. A. Loss of Head Between Conduit and Port in Divided Flow

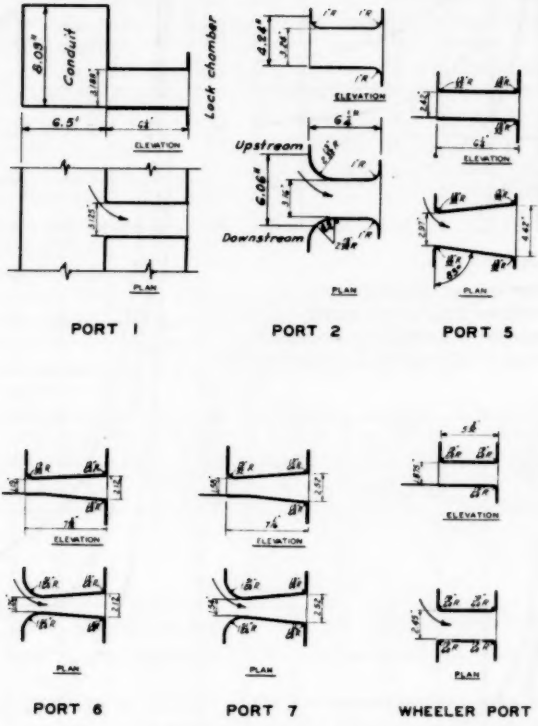


Fig. B. Port Designs

for the conditions noted above, the value of $Q_p/Q = 0.7$ was obtained from a graph of Eq. (4). For greater values of Q_p/Q the port should flow full and head loss based on the contraction coefficient will be zero. For lower values of Q_p/Q the jet would be contracted in the vertical but not in plan until the maximum effective width can be used.

According to the theoretical analysis the Wheeler port will have a contracted jet smaller than the minimum port area throughout the range of values of Q_p/Q, thus the theoretical curves remain above the value for h'_f = zero. Similar conditions exist for square-edged port No. 1.

Data for port 5 are presented because the greatest range of the head loss ratio was measured in tests on this port. However, port 5 is probably the least practical of the four. It was assumed that the barrier in the plan view was of sufficient length to deflect the jet at an angle of 83 degrees. Accordingly a plot was made of Eq. (4) for the special case of the jet deflected 83 degrees. However, for the range of available test data i.e., up to $Q_p/Q = 0.7$ it was found that values of C_c were increased by less than 1 percent over corresponding values for a 90-degree jet. Values of C_c applicable to port 5 would probably vary from those for a 90-degree jet by less than 0.5 percent.

Fig. A shows that only for port 5 do the head losses fall substantially above the midordinate line. For ports 1, 5, and Wheeler port there was some consistency in the relationship between the size of the jet and the outlet area of the port at the point where the head loss dropped below the midordinate curve.

Flared ports 6 and 7 were more efficient than port 5. Flared 9 and 10.5 degrees, respectively, these ports indicated head losses for values of Q_p/Q greater than 0.15 which were not in excess of the loss that might be expected in gradual expansions twice as great.

SADIQ M. NIAZ,[1] J. M. ASCE— As a part of the study outlined by the author, the writer made detailed observations on certain phases of converging manifold flow. In particular, information was obtained concerning the terms for the head loss and the unbalanced force for this type of flow. These observations together with alternative forms of plotting are presented herein as an aid in the interpretation of the data.

From application of a simplified energy relationship the author obtained Eq. (6):

$$\frac{\Delta h}{V^2/2g} = 1 - \left(\frac{V_c}{V'}\right)^2 + \frac{h_f}{V^2/2g} \qquad (6)$$

Because the conduit velocity increases at the junction, the loss experienced by the conduit flow is somewhat similar to that for flow through a boundary contraction. It can, logically, be expressed in terms of loss coefficients to be applied either to the velocity head of approach or to the difference of the upstream and downstream velocity heads; the two are simply related through the discharge ratio and the diameter ratio. Thus, from the latter alternative, the head loss can be expressed in the form:

$$h_f = m\left(\frac{V^2 - V_c^2}{2g}\right) = m\frac{V^2}{2g}\left(1 - \frac{V_c^2}{V^2}\right) \qquad (a)$$

1. Director Central Designs, Punjab Irrigation Dept., Lahore, Pakistan.

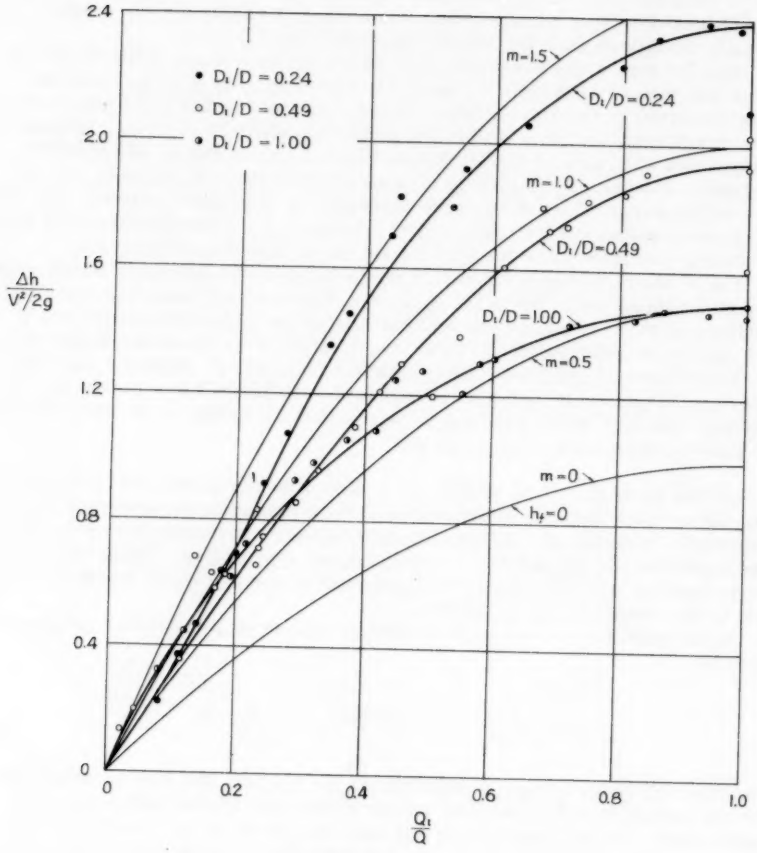

2.4
2.0
1.6
1.2
0.8
0.4
0
0.2
0.4
0.6
0.8
1.0
Δh / V²/2g
Q_t / Q
• D_t/D = 0.24
○ D_t/D = 0.49
• D_t/D = 1.00
m=1.5
D_t/D=0.24
m=1.0
D_t/D=0.49
D_t/D=1.00
m=0.5
m=0
h_f=0

Figure A

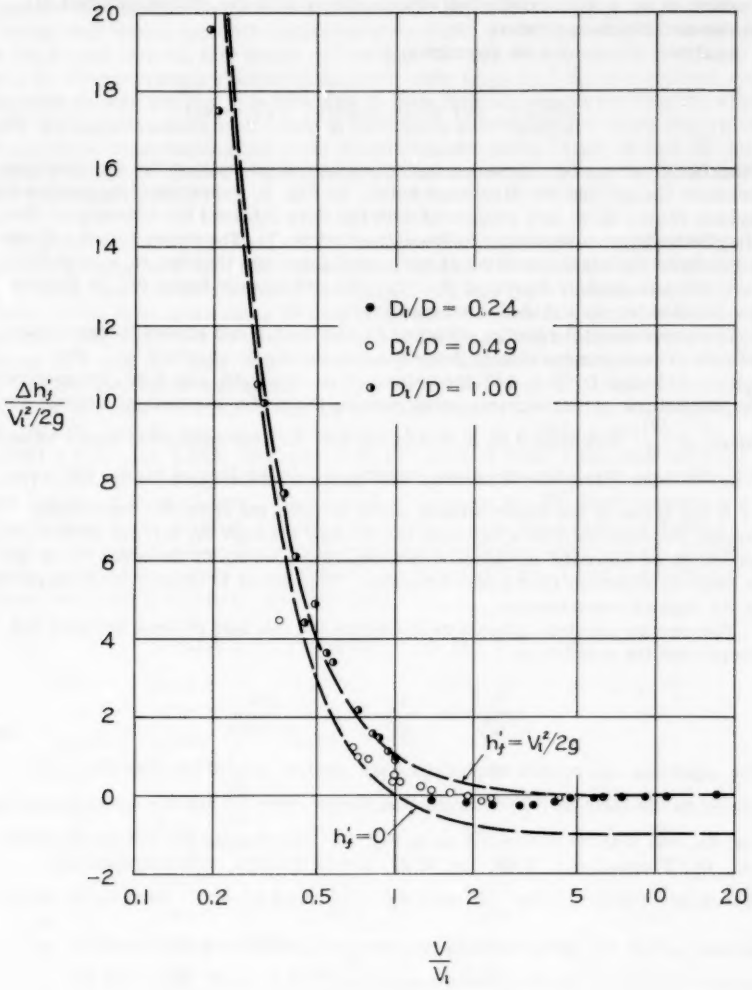

20
18
16
14
12
10
8
6
4
2
0
-2
$\frac{\Delta h_f'}{V_t^2/2g}$
$D_t/D = 0.24$
$D_t/D = 0.49$
$D_t/D = 1.00$
$h_f' = V_t^2/2g$
$h_f' = 0$
0.1
0.2
0.5
1
2
5
10
20
$\frac{V}{V_t}$

Figure B

in which m is a loss coefficient which varies with the aforementioned diameter and discharge ratios.

Equation (6) can now be rewritten:

$$\frac{\Delta h}{V^2/2g} = \left[2\frac{Q_l}{Q} - \left(\frac{Q_l}{Q}\right)^2\right](1+m) \qquad \text{(b)}$$

A family of curves for various constant values of m relates the dimensionless pressure change and the discharge ratio. In Fig. A, representative curves for various values of m are compared with the data obtained for convergent flow, this plot being an alternative to the author's Fig. 7. The curve for m = O corresponds to the ideal condition of zero head loss, and that for m = 1 to the condition assumed by Favre of F = O. The unbalanced force (F) is directed downstream for $m < 1$ and upstream for $m > 1$.

The experimental results, obtained by the writer and quoted by the author, indicate trends comparable to those for the curves of constant m. For $D_l/D = 1/4$ and $D_l/D = 1/2$ the values of m are 1.38 and 0.95, respectively. The departure of the experimental curves from the theoretical curves for values of $\frac{Q_l}{Q}$ less than 0.25 is due to the fact that for relatively small values of Q_l the type of mixing occurring results in a lower loss of head. For $\frac{D_l}{D}$ = 1.0 the trend of the experimental curve is different from the theoretical curves, because the water entering the conduit through the lateral always experiences an increase in velocity; consequently, jet action does not occur and an entirely different flow pattern results. This trend is equally well supported by the Munich experiments.

The energy equation gives a relationship for the loss of head between the lateral and the conduit as

$$\frac{h_e'}{V_l^2/2g} = \frac{V^2}{V_l^2} - 1 + \frac{\Delta h'}{V_l^2/2g} \qquad \text{(d)}$$

The experimental results correlating the change in head for flow from the lateral to the conduit (h_f') with the discharge ratio $\frac{Q_l}{Q}$ (shown in the author's Fig. 8), can also be presented as in Fig. B. The results for the three cases, i.e., D_l/D equal to 1, 0.49, and 0.24, are presented. The two limiting curves are defined by Eq. (d) with $h_f' = \frac{V_l^2}{2g}$ and $h_f' = O$. Most of the experimental points lie between these two curves, particularly for values of $\frac{V_l}{V}$ greater than 0.30 (which corresponds to $\frac{Q_l}{Q}$ = 0.3, 0.07, and 0.02 for $\frac{D_l}{D}$ = 1.0, 0.49, and 0.24, respectively). The points for the three cases have a tendency to form one continuous curve if plotted in this way. For higher values of $\frac{V_l}{V}$, the values of $\frac{\Delta h'}{V_l^2/2g}$ are very nearly zero. For values of $\frac{V_l}{V}$ greater than 1.0, $\frac{\Delta h'}{V_l^2/2g}$ is negative, a point discussed in some detail by the author.

The pressure measurements along the sides of the lateral just before the junction showed that the pressure on one side differed from that on the other side by very small amounts (approximately 0.005 ft. of mercury) in the case of the 1-inch lateral; this bears out the fact that for this case and $F \approx 0$ or $m \approx 1$. These pressure differences were quite large (0.1 ft. of mercury) and opposite in sign for the 1/2-inch and 2-inch laterals, thus verifying the existence of a sizable unbalanced force F in the two directions for these two cases. From these observations and from the alternative plots (Figs. A and B), additional insight is gained in the interpretation and evaluation of the unbalanced force and head losses.

RALPH W. POWELL,[1] M. ASCE—Despite the many papers that have been written on manifold flow, the subject has remained in a somewhat unsatisfactory state, and this treatment of the subject and the report on the results obtained at the Iowa Institute of Hydraulic Research is most welcome. The writer wishes however to call attention to one important mass of data on combining flow that had apparently not come to the author's attention.

In 1931 Naramoto and Kasai[2] reported on a series of 651 runs on combining flow in cylindrical pipes with intersection angles of 15, 30, 45, 60, 70, 75, 80, 85, and 90 degrees, and ratios of D_1 to D of 1.004, 0.815, 0.678, 0.666, 0.581, and 0.503. Unfortunately for most of these runs only the total loss of energy is given, without telling how much of it was in the main pipe, and how much in the lateral. But for 29 of the runs for 90° intersections the complete data are printed, and from these the writer has prepared Tables 1 and 2. In general they show good agreement with the data reported in this paper, most of the points lying between the lines in Figs. 7 and 8 which represent the Iowa and Munich experiments.

TABLE 1 - Kyushu data with $D_1/D = 1.004$

Run	Q_1/Q	$2g\,\Delta h/V^2$	$2g\,h_f'/V^2$	$2g\,h_f'/V_1^2$
1	0.135	0.450	- 0.480	- 26.7
2	0.233	0.627	- 0.261	- 4.84
3	0.341	0.939	0.100	0.82
4	0.409	1.137	0.404	2.45
5	0.484	1.161	0.368	1.59
6	0.528	1.218	0.448	1.63
7	0.564	1.268	0.510	1.64
8	0.602	1.314	0.580	1.62
9	0.653	1.352	0.648	1.54
10	0.690	1.390	0.699	1.49
11	0.704	1.396	0.684	1.43
12	0.728	1.425	0.760	1.47
13	0.745	1.447	0.808	1.48
14	0.760	1.467	0.841	1.47
15	0.767	1.453	0.846	1.4^
16	0.774	1.461	0.878	1.46

1. Prof. of Mechanics, The Ohio State Univ., Columbus, Ohio
2. "On the Loss of Energy at Impact of Two Confined Streams of Water" by Itaru Naramoto and Taijiro Kasai, Memoirs of the College of Engineering, Kyushu Imperial University, Vol. VI, No. 3, Fukuoka, Japan, 1931.

TABLE 2 - Kyushu data with $D_1/D = 0.503$

Run	Q_1/Q	$2g\,\Delta h/V^2$	$2g\,h_f'/V^2$	$2g\,h_f'/V_1^2$
1	0.135	0.396	- 0.184	- 0.642
2	0.208	0.633	0.227	0.333
3	0.270	0.797	0.656	0.574
4	0.322	0.914	1.018	0.624
5	0.368	1.043	1.442	0.680
6	0.399	1.120	1.727	0.692
7	0.435	1.193	2.085	0.703
8	0.472	1.268	2.548	0.729
9	0.509	1.132	2.990	0.734
10	0.522	1.386	3.271	0.766
11	0.567	1.496	3.813	0.756
12	0.605	1.545	4.385	0.764
13	0.598	1.543	4.294	0.765

JOHN K. VENNARD[1] A. M. ASCE, and DARRHL I. DENTONI,[2] J. M. ASCE—Congratulations to Professor McNown for his thorough analytical and experimental study of the manifold problem for simple basic geometries of the conduit junctions. The writers have recently been involved in experimental work of a similar nature and are thus in a position to appreciate the excellent and concise presentation of theories and facts in a field which seems to have all too many "loose ends."

Since the completion of the thesis by James R. Barton [1], the senior writer has been intrigued by an apparently serious discrepancy between the results obtained at Iowa and at Munich for the case of dividing flow. This discrepancy is not evident on Professor McNown's Fig. 4, in fact he makes the point that "the two series of experiments [i.e., Iowa and Munich] indicate almost identical results." From study of the Munich papers and comparison with Professor McNown's interpretation of these on his Fig. 4, the writers believe that the Munich curves have been mislabeled and the results are not only not "identical" but exhibit a trend opposite to those obtained at Iowa. This apparent discrepancy led to the junior writer undertaking an experimental program in an attempt to resolve the conflicting results obtained from these two carefully performed sets of experiments. Accordingly tests were conducted in the Stanford Hydraulic Laboratory on five square-edged outlets having axes at 90° to a main conduit of 1.276 inch diameter. Lateral sizes were selected which gave diameter ratios D_1/D of 0.294, 0.392, 0.490, 0.642, and 0.830 and the results plotted in the manner suggested by Professor McNown on his Figs. 4 and 5. Good agreement with the Iowa and Munich results was obtained in the case of Fig. 5 and fair agreement in that of Fig. 4.

1. Professor of Fluid Mechanics, Stanford University, Stanford, California.
2. Junior Engineer, Davis and Holman, Consulting Engineers, Stockton, California.

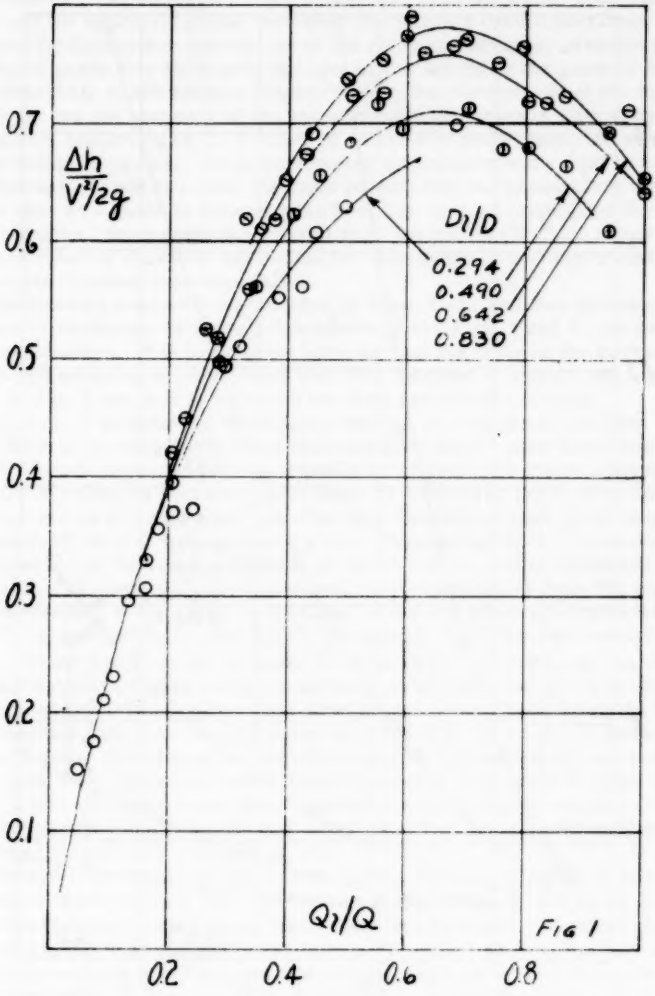
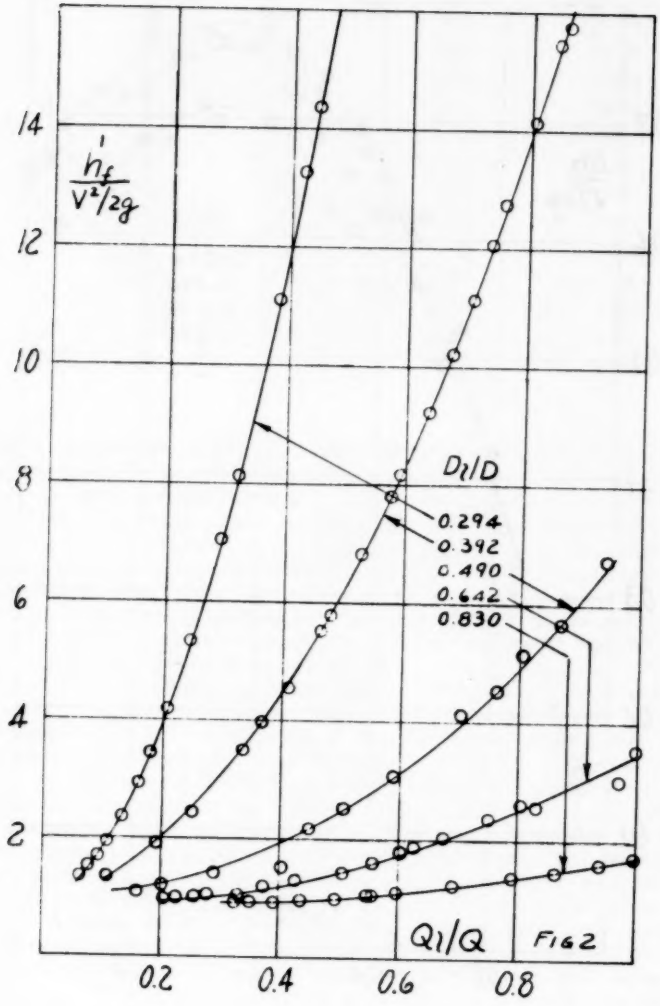

0.7
0.6
0.5
0.4
0.3
0.2
0.1
Δh / V²/2g
D1/D
0.294
0.490
0.642
0.830
Q1/Q
0.2
0.4
0.6
0.8

FIG 1

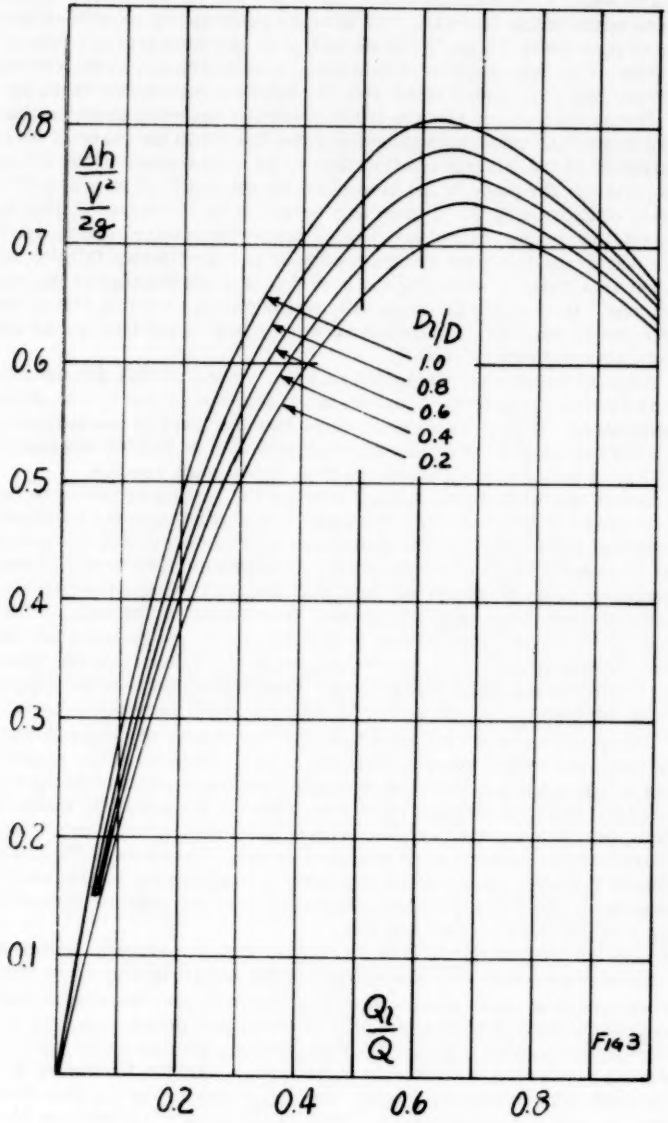

14
12
10
8
6
4
2
$\frac{h_f'}{V^2/2g}$
D_1/D
0.294
0.392
0.490
0.642
0.830
0.2
0.4
0.6
0.8
Q_1/Q
FIG 2

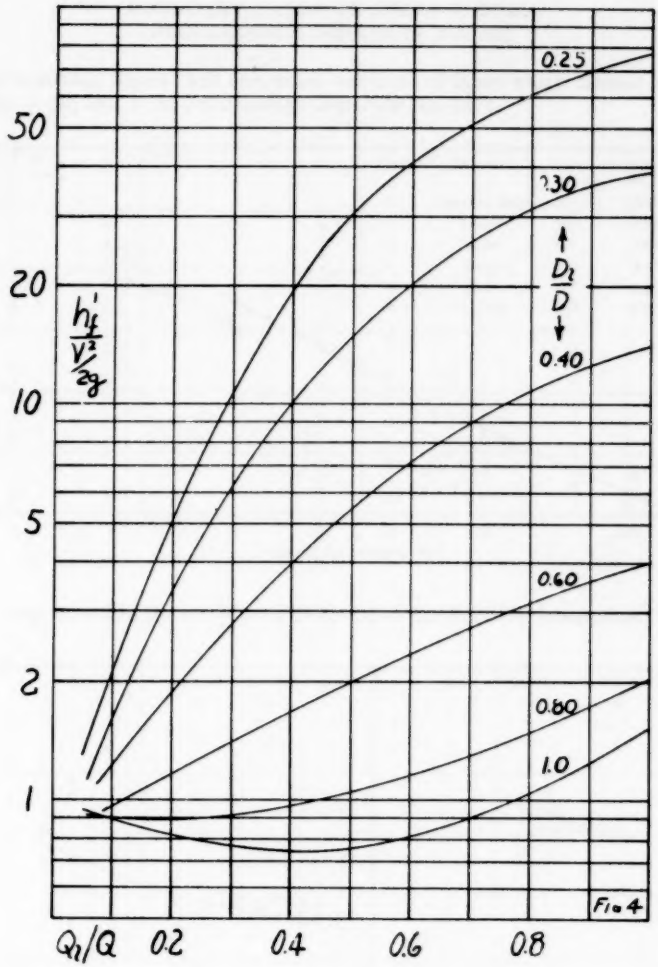
The Stanford tests were carried out in the same general manner as those at Iowa and Munich but differed in the one detail that no piezometer connections were made in the laterals. The laterals consisted of smooth straight sections of pipe some 27 to 30 diameters in length discharging freely to the atmosphere. The diameters were measured (volumetrically) with extreme care and the hydraulic grade lines over the laterals established from the Blasius friction factor and the use of the generally accepted premise that the hydraulic grade line intersects the axis of the lateral in the plane of its outlet. The reliability of the Blasius friction factor, the correctness of the foregoing premise, and the adequacy of the above-mentioned length of lateral had been accurately confirmed by C. T. Chen in a series of tests connected with another experimental program. Many piezometer connections were used to establish the hydraulic grade line over the main conduit and the Blasius friction factor again used as a guide in selecting the best line through the tops of the piezometer columns. Here again the reliability of the Blasius friction factor was demonstrated by excellent agreement between a line of predetermined slope and points obtained experimentally.

The Stanford results are presented in Figs. 1 and 2 of this discussion and, in view of Professor McNown's discussion of his Figs. 4 and 5, are presumably self-explanatory. It is to be noted however that the trend of the curves confirms that obtained at Iowa rather than that obtained at Munich and that the results of Fig. 2 are in line with both the Iowa and Munich results.

The lack of quantitative agreement between the results of the three test programs is not surprising since the quantities depend upon the relatively small vertical distances between extensions of hydraulic grade lines which are difficult to establish with great accuracy. It appears to the writers however that agreement is good enough and data now plentiful enough to develop a set of "standard values" for engineering use. Proceeding on the assumption that the accuracy of the results obtained at Munich, Iowa, and Stanford are about the same, "cross-plots" were prepared from points taken from the smooth curves obtained at the three institutions. This was accomplished by plotting $\Delta h/V^2/2g$ against D_1/D and $h_f'/V^2/2g$ against D_1/D for each even 0.1 of Q_1/Q. These plots are not included here but the scatter of points on the first set was large and on the second relatively small. Because of the scatter it was judged that nothing better than straight lines were justified in the first set of crossplots; the best straight lines were fitted to the points by the method of least squares. For the second set of crossplots where scatter was less, curves were quite well defined and were drawn in by eye. The plots of Figs. 3 and 4 were finally obtained; since these represent a composition and reconciliation of results from three separate test programs, they are offered as tentative "standard values" for engineering use.

Although the writers do not wish to quibble over the symbols selected by Professor McNown, they feel that the use of the terms h_f and h_f' to symbolize local head losses is unfortunate since these symbols give the casual reader the impression that they embody some sort of "friction" phenomena. As an alternative the writers would suggest that the upstream portion of the main conduit be designated by 1, the downstream portion by 2, and the lateral by 3. The term h_f could then be replaced by h_{12} (or $h_{L_{12}}$) and h_f by h_{13} (for dividing flow) and h_{32} (for combining flow). This would seem to be more in line with generally accepted standard symbols.


0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Δh / V²/2g
D1/D
1.0
0.8
0.6
0.4
0.2
Q1/Q
F143
0.2
0.4
0.6
0.8


50
20
10
5
2
1
$\frac{h_f'}{V^2/2g}$
0.25
0.30
$\frac{D_1}{D}$
0.40
0.60
0.80
1.0
Fig. 4
Q_1/Q
0.2
0.4
0.6
0.8

CORRECTIONS TO THE DEVELOPMENT OF THE TURBULENT BOUNDARY LAYER ON STEEP SLOPES PROCEEDINGS-SEPARATE NO. 281

The following figures are corrected versions of those which appear as Figures 7, 10, and 11 of Proceedings-Separate No. 281:

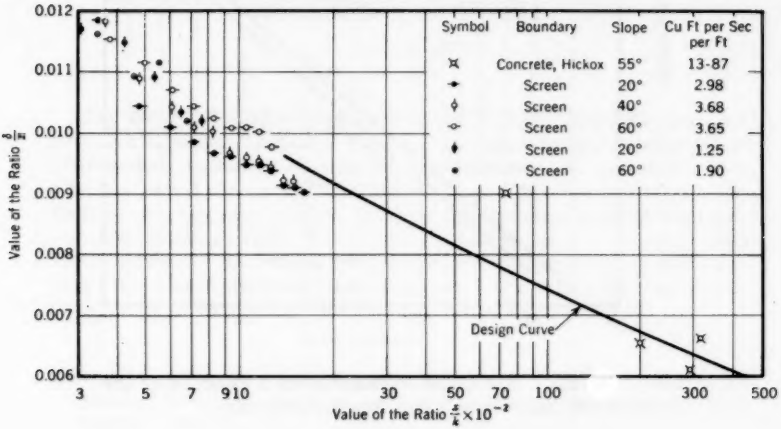


Fig. 7. - The Ratio $\frac{\delta}{x}$ as a Function of the Relative Roughness.

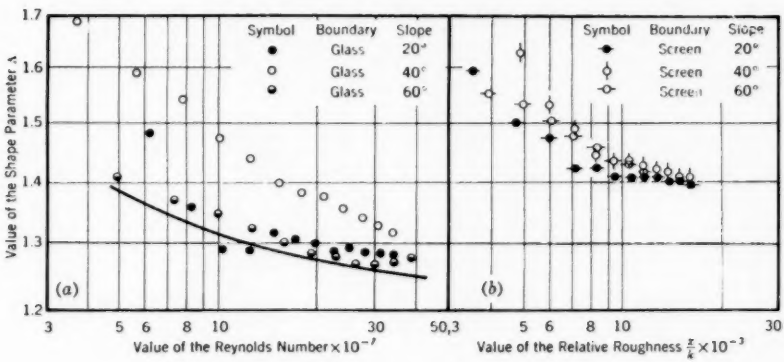


Fig. 10. - Graphical Solution of the Illustrative Problem.

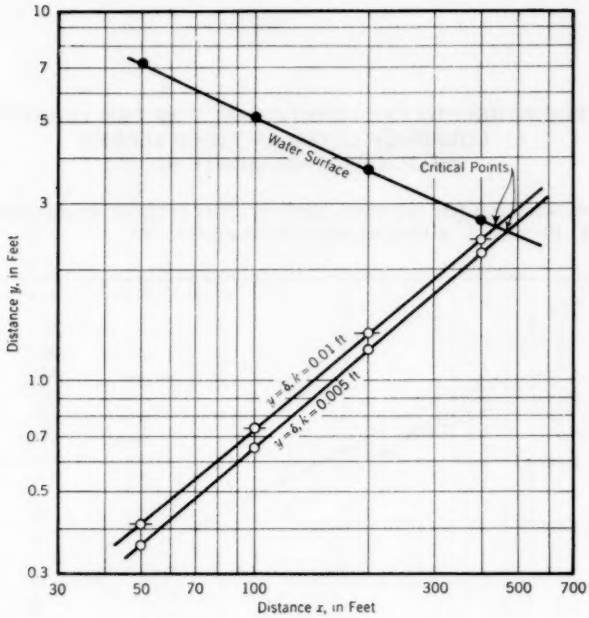


Fig. 11. - The Shape Parameter as a Function of the Reynolds Number and Relative Roughness.

PROCEEDINGS-SEPARATES

VOLUME 80 (1954)

The technical papers published in the current calendar year are presented below. Technical division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering."

JANUARY: 379(SM),[a] 380(HY), 381(HY), 382(HY), 383(HY), 384(HY),[a] 385(SM), 386(SM), 387(EM), 388(SA), 389(SU),[a] 390(HY), 391(IR),[a] 392(SA), 393(SU), 394(AT), 395(SA),[a] 396(EM),[a] 397(ST).[a]

[a]Discussions grouped by Divisions.

AMERICAN SOCIETY OF CIVIL ENGINEERS

OFFICERS FOR 1954

PRESIDENT

DANIEL VOIERS TERRELL

VICE-PRESIDENTS

Term expires October, 1954:
EDMUND FRIEDMAN
G. BROOKS EARNEST

Term expires October, 1955:
ENOCH R. NEEDLES
MASON G. LOCKWOOD

DIRECTORS

Term expires October, 1954:
WALTER D. BINGER
FRANK A. MARSTON
GEORGE W. McALPIN
JAMES A. HIGGS
I. C. STEELE
WARREN W. PARKS

Term expires October, 1955:
CHARLES B. MOLINEAUX
MERCEL J. SHELTON
A. A. K. BOOTH
CARL G. PAULSEN
LLOYD D. KNAPP
GLENN W. HOLCOMB
FRANCIS M. DAWSON

Term expires October, 1956:
WILLIAM S. LaLONDE, JR.
OLIVER W. HARTWELL
THOMAS C. SHEDD
SAMUEL B. MORRIS
ERNEST W. CARLTON
RAYMOND F. DAWSON

PAST-PRESIDENTS

Members of the Board

CARLTON S. PROCTOR
WALTER L. HUBER

TREASURER

CHARLES E. TROUT

EXECUTIVE SECRETARY

WILLIAM N. CAREY

ASSISTANT TREASURER

GEORGE W. BURPEE

ASSISTANT SECRETARY

E. L. CHANDLER

PROCEEDINGS OF THE SOCIETY

HAROLD T. LARSEN
Manager of Technical Publications

DEFOREST A. MATTESON, JR.
Editor of Technical Publications

PAUL A. PARISI
Assoc. Editor of Technical Publications

COMMITTEE ON PUBLICATIONS

FRANK A. MARSTON

I. C. STEELE
GLENN W. HOLCOMB
ERNEST W. CARLTON
OLIVER W. HARTWELL
SAMUEL B. MORRIS